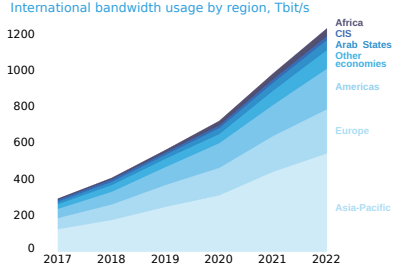
Introducing RSESS

An Open-Source Enumerative Sphere Shaping Implementation Coded in Rust

Frederik Ritter, Andrej Rode, and Laurent Schmalen

Why Probabilistic Shaping?

- Consistent growth of internet usage drives demand for higher bandwidth on fiber optical networks **[ITU22]**



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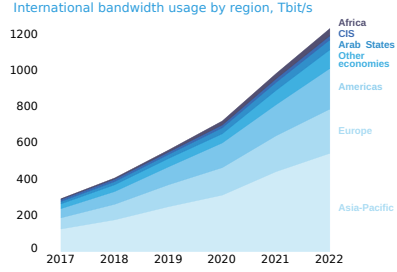

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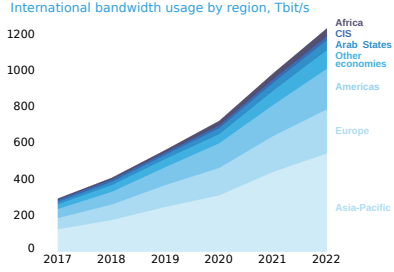

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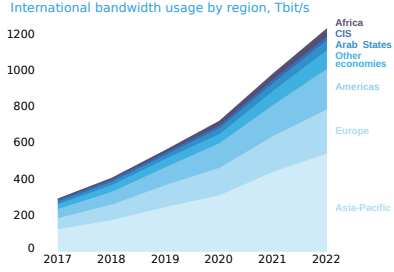
- **Our contribution:** A practical implementation of a probabilistic shaping algorithm for research and development



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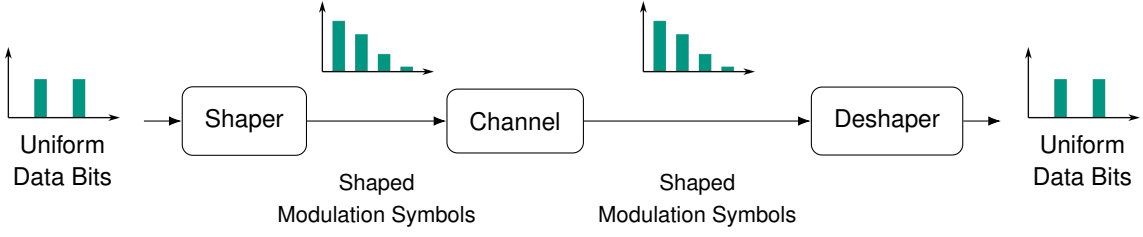

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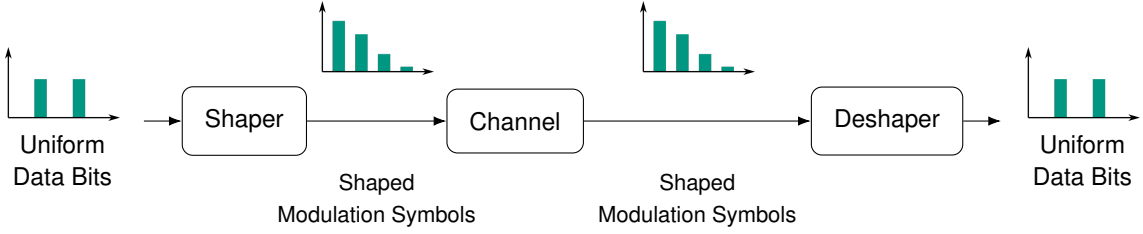

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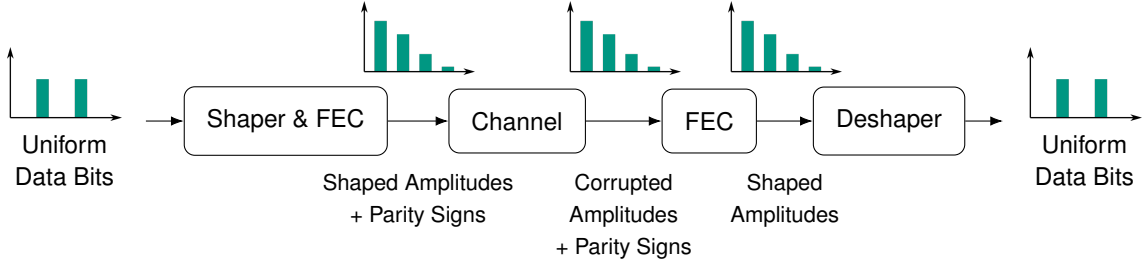

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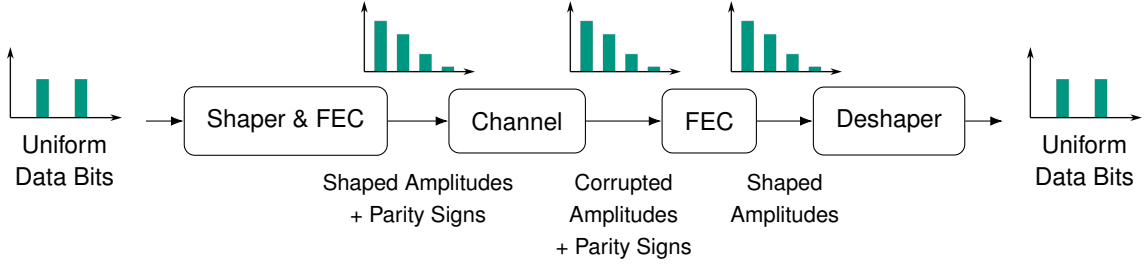

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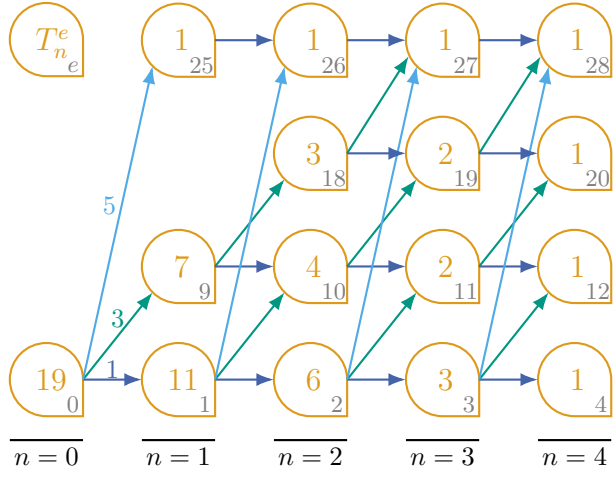

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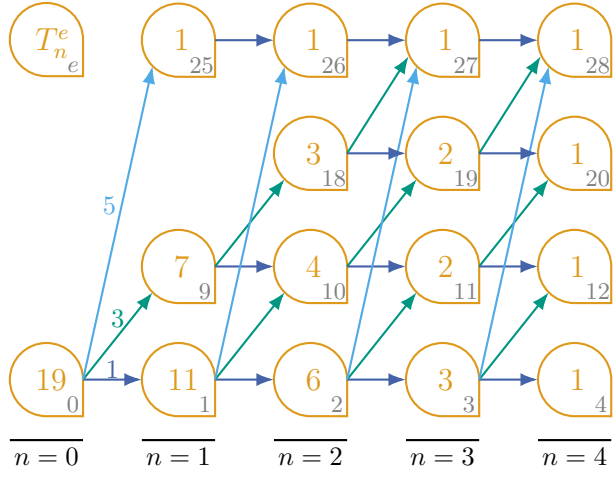

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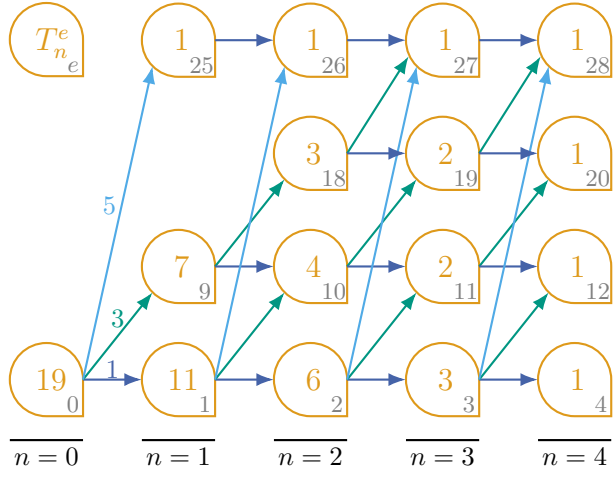

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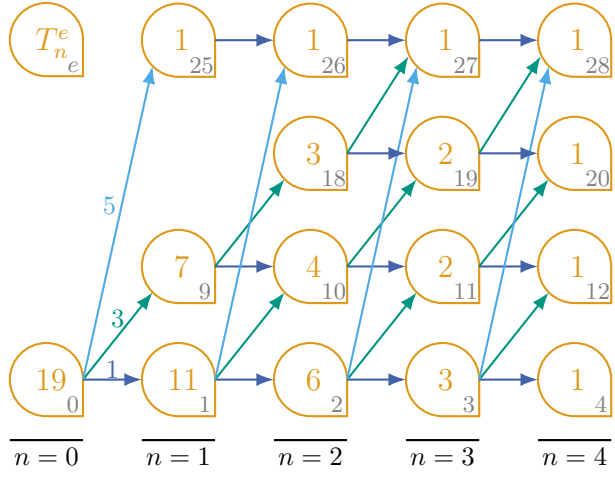

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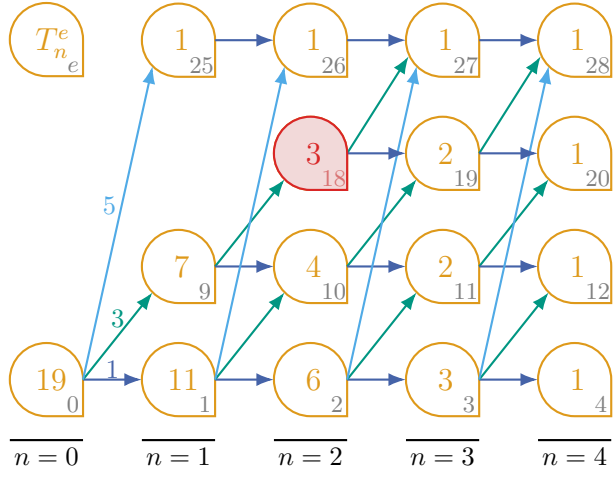

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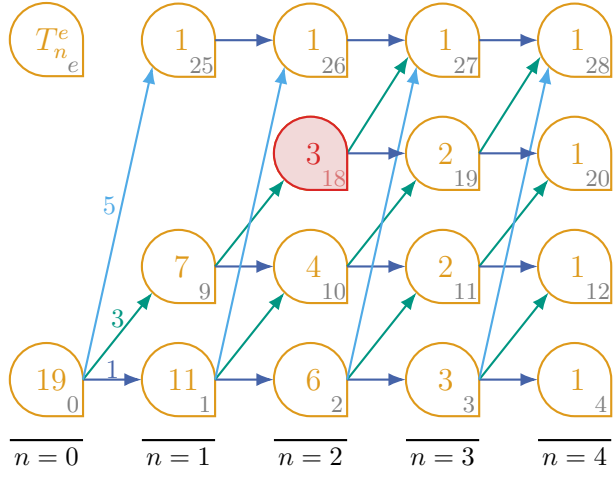

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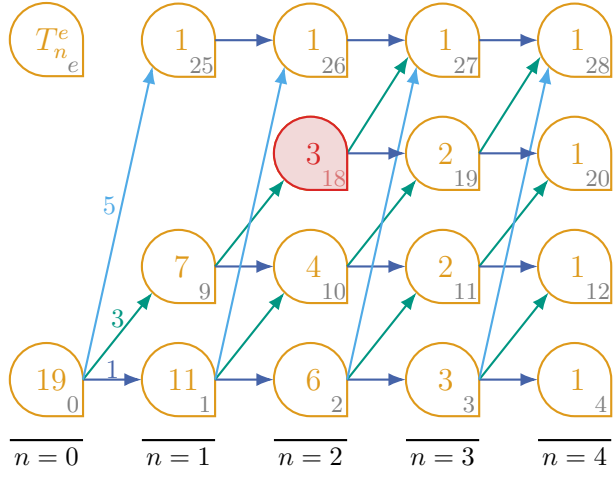

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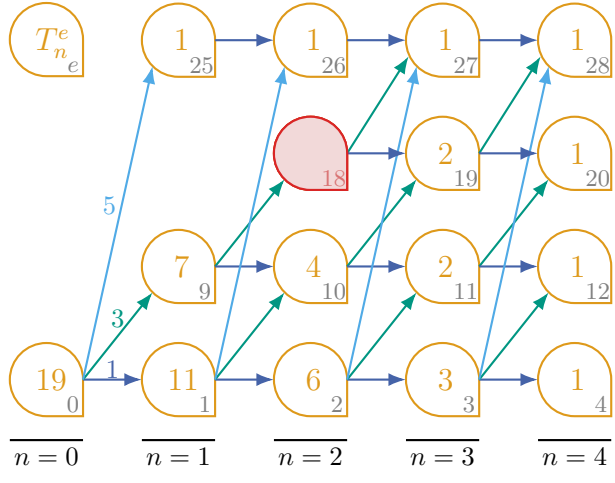

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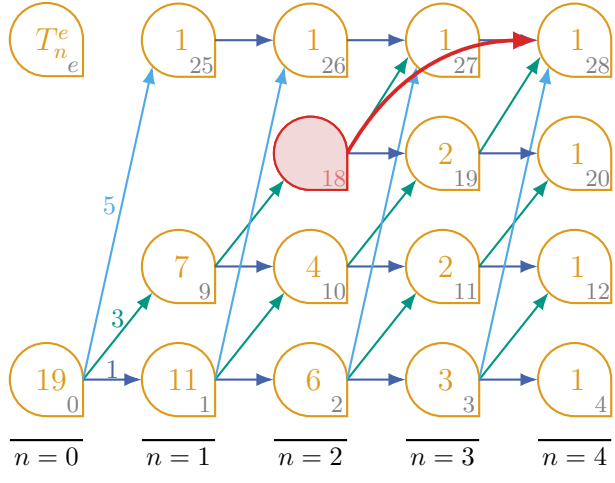

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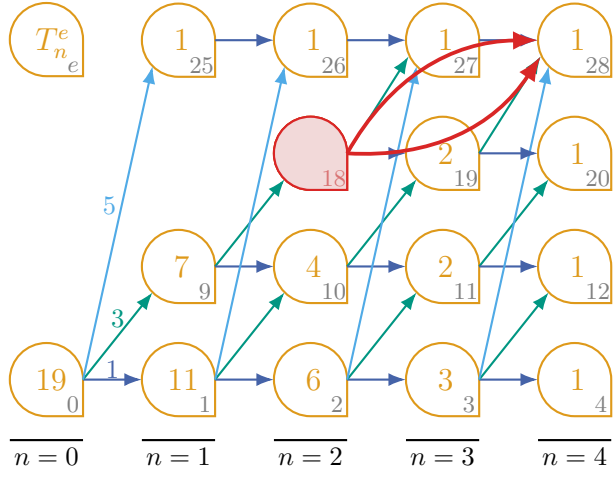

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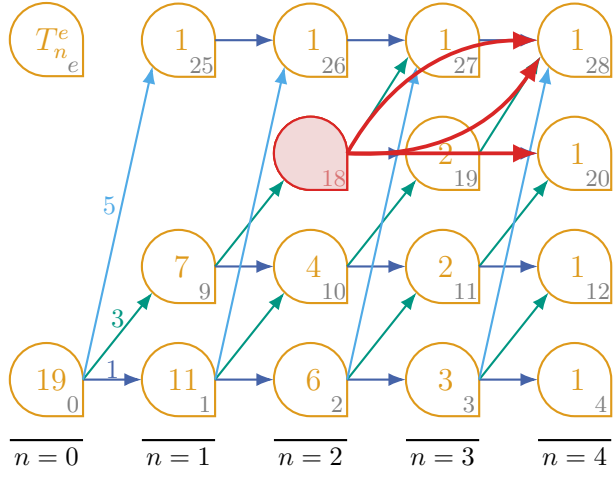

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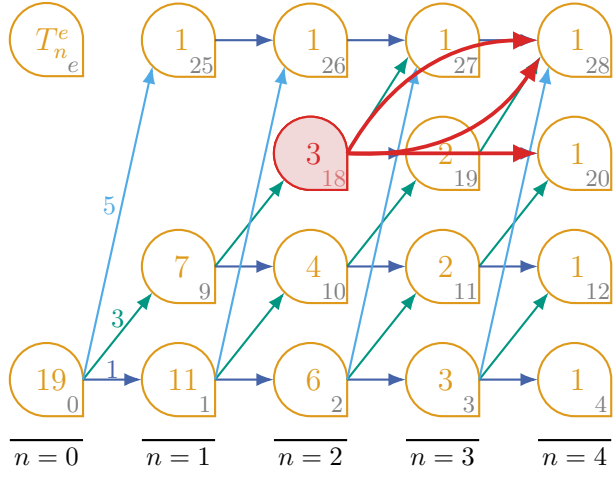

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github.com
/kit-cel/rsess

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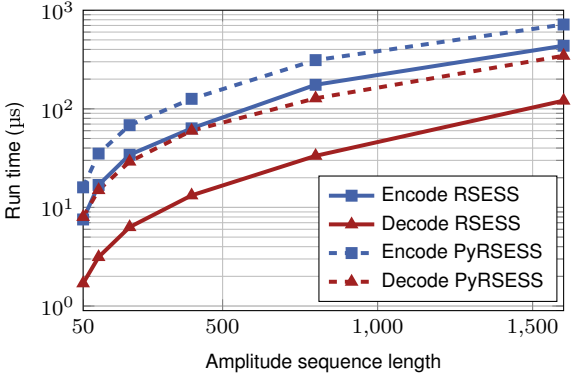

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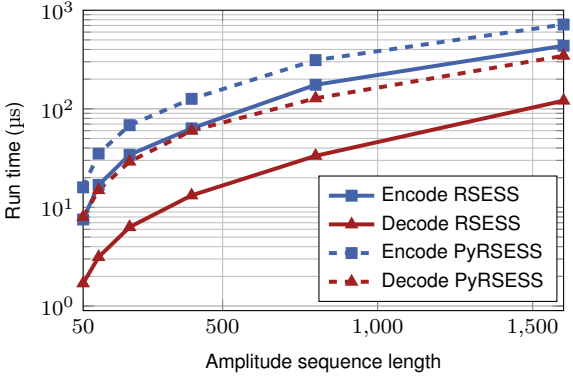

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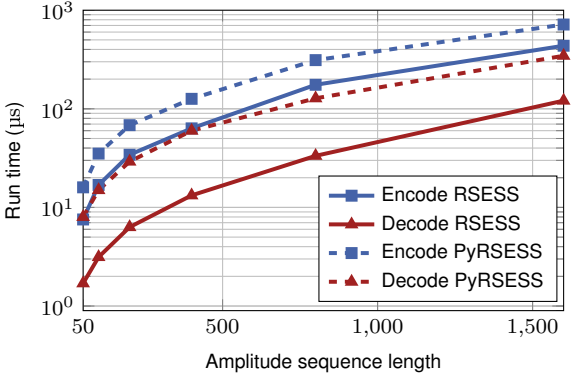
- Shaping rate (number of bits per amplitude) held constant at 1.5
- Tested on a 4-core Intel Core i7-7700 (2017)



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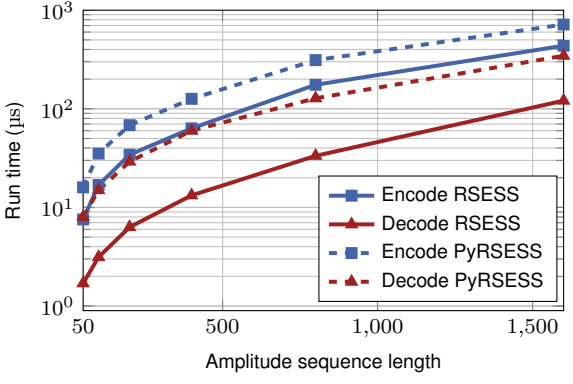

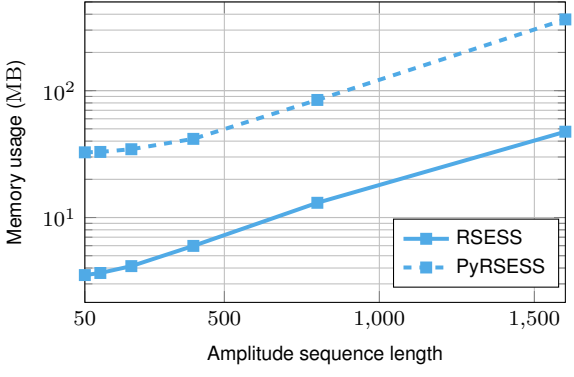

RSESS Validation

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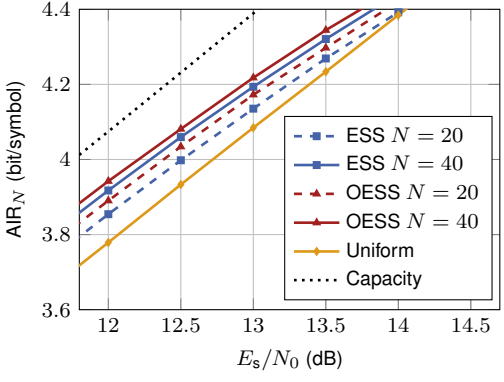
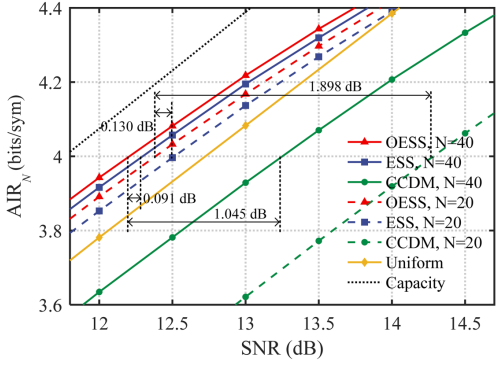
Results on AWGN channel with RSESS



Reference results in [CC22]



[CC22] Y. Chen and J. Chen and W. Li and M. Zhang and D. Liu and M. Tang, "On optimization and analysis of enumerative sphere shaping for short blocklengths," *Journal of Lightwave Technology*, vol. 40, pp. 7265-7278, Nov. 2022.

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Questions?

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