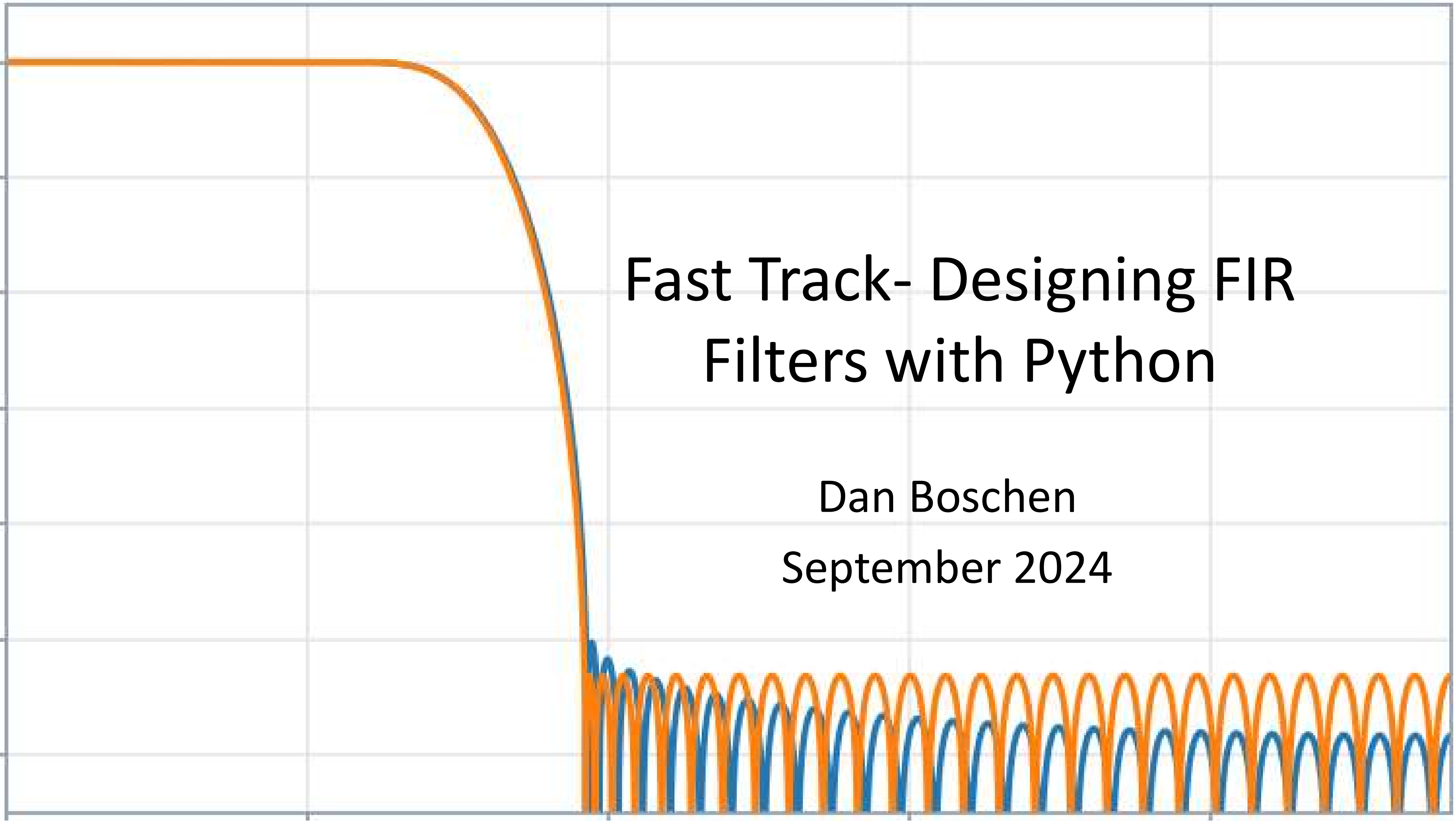
Fast Track- Designing FIR Filters with Python



Dan Boschen

September 2024

Content is derived from courses on DSP and Python taught by Dan Boschen

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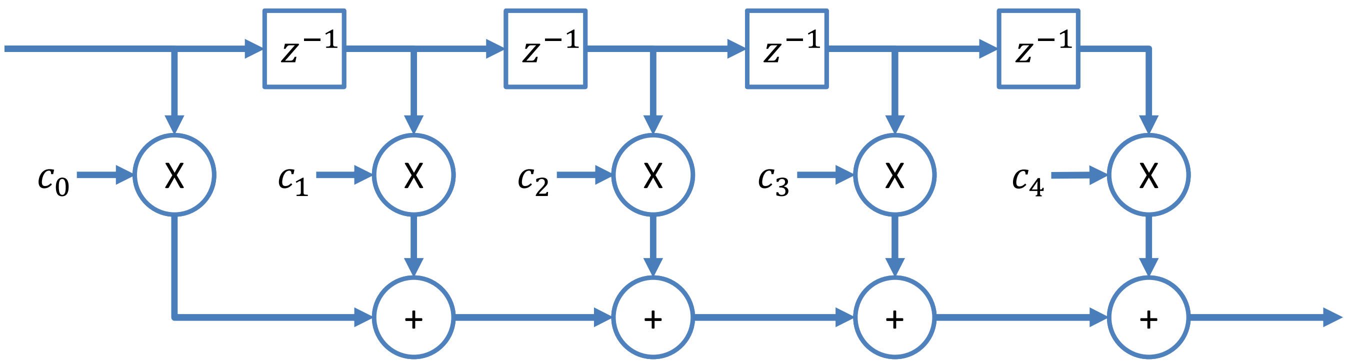
FIR Filters



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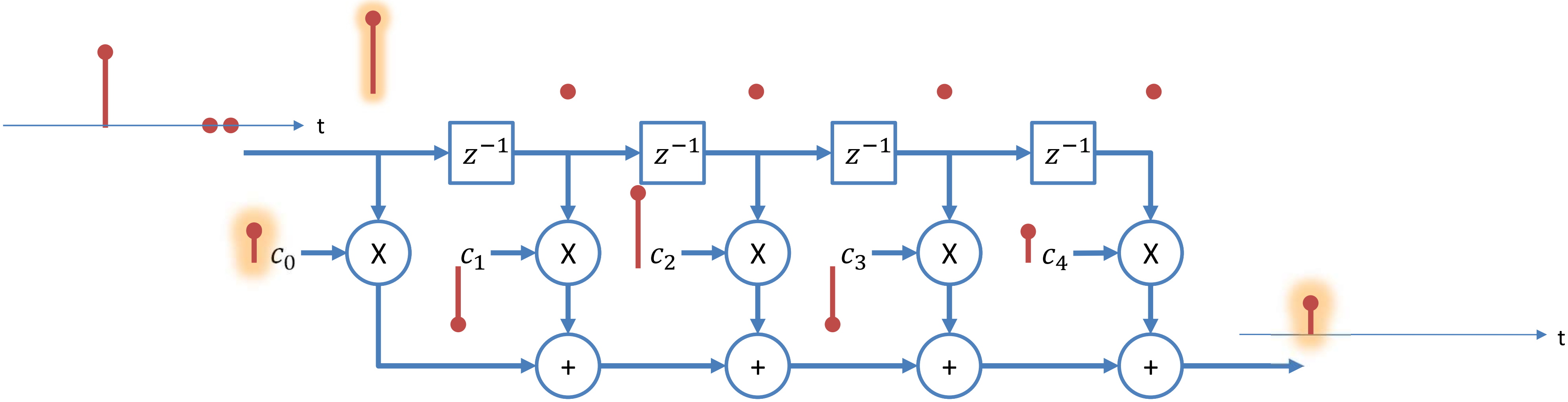
FIR Filters



The Coefficients of the Filter is the **Impulse Response**...

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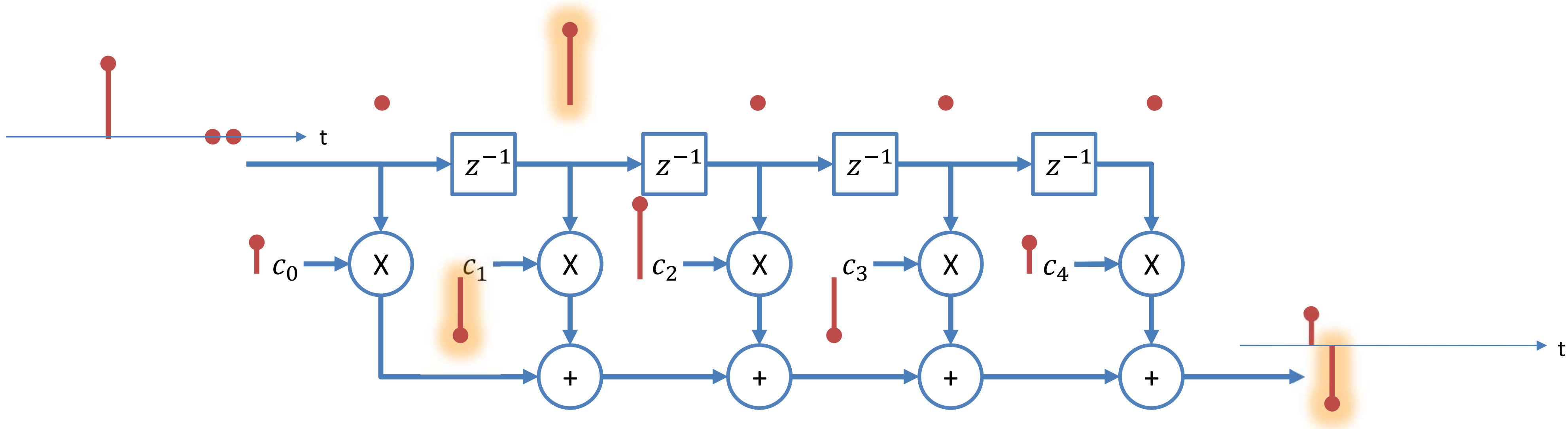
FIR Filters



The Coefficients of the Filter is the **Impulse Response**...

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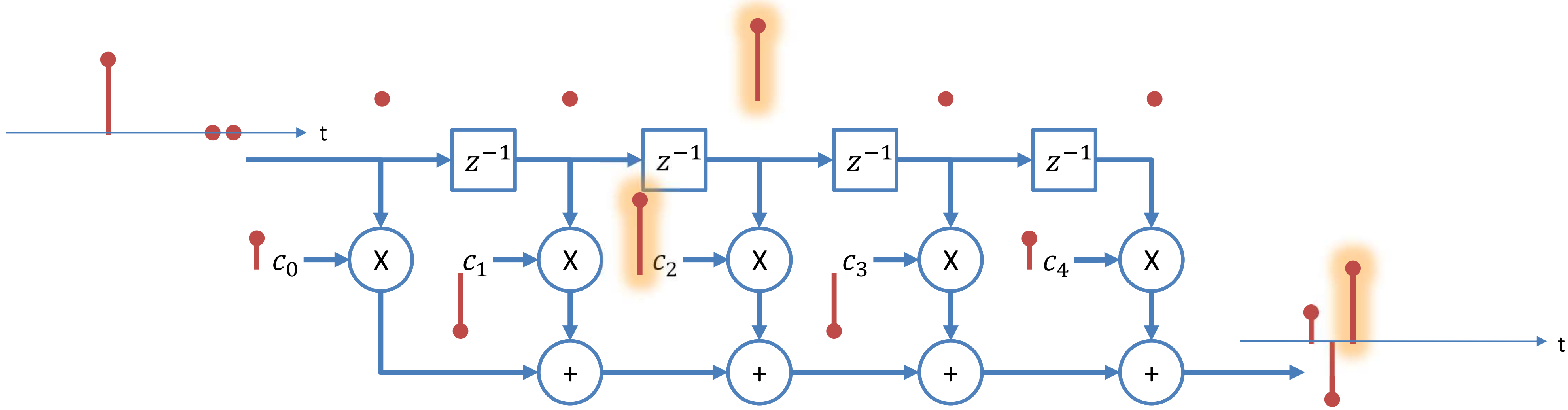
FIR Filters



The Coefficients of the Filter is the **Impulse Response**...

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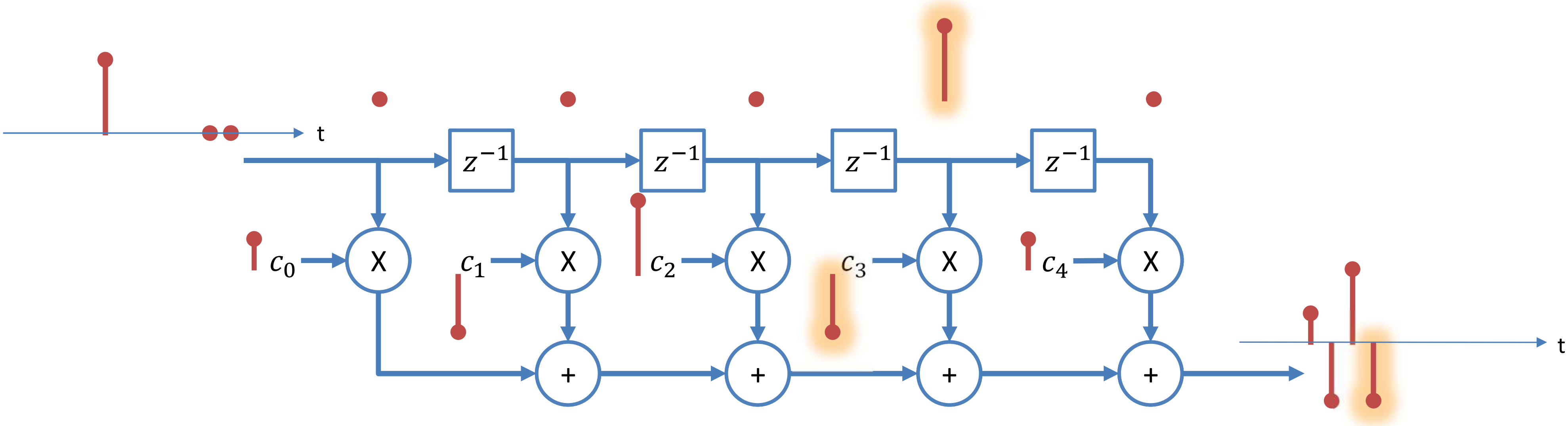
FIR Filters



The Coefficients of the Filter is the **Impulse Response**...

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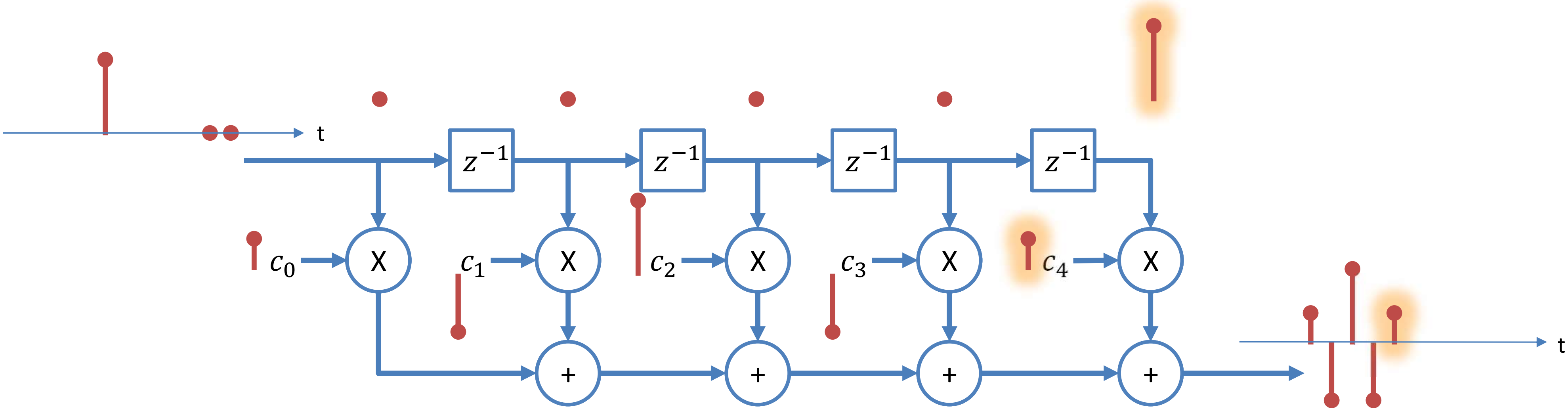
FIR Filters



The Coefficients of the Filter is the **Impulse Response**...

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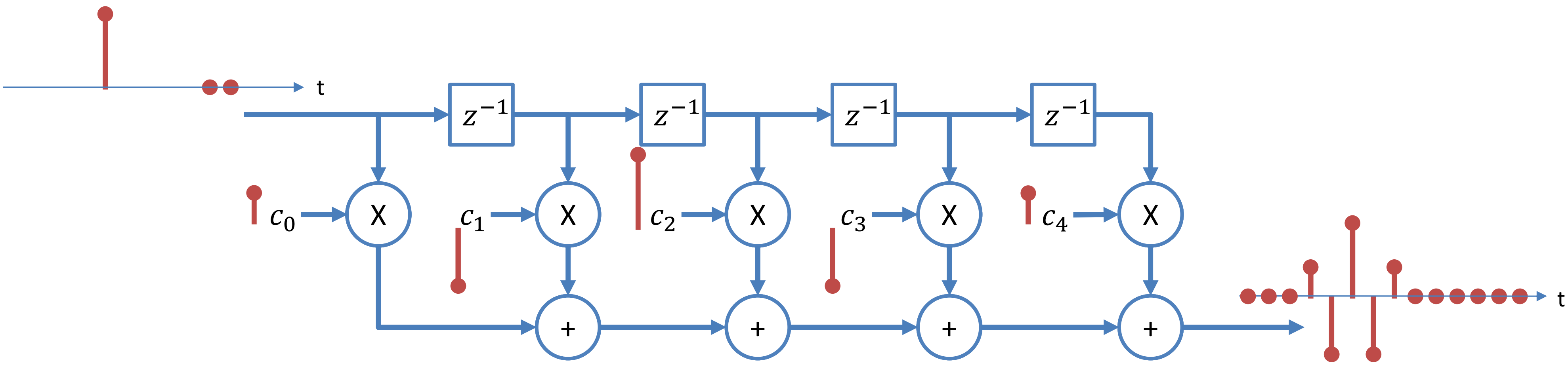
FIR Filters



The Coefficients of the Filter is the **Impulse Response**...

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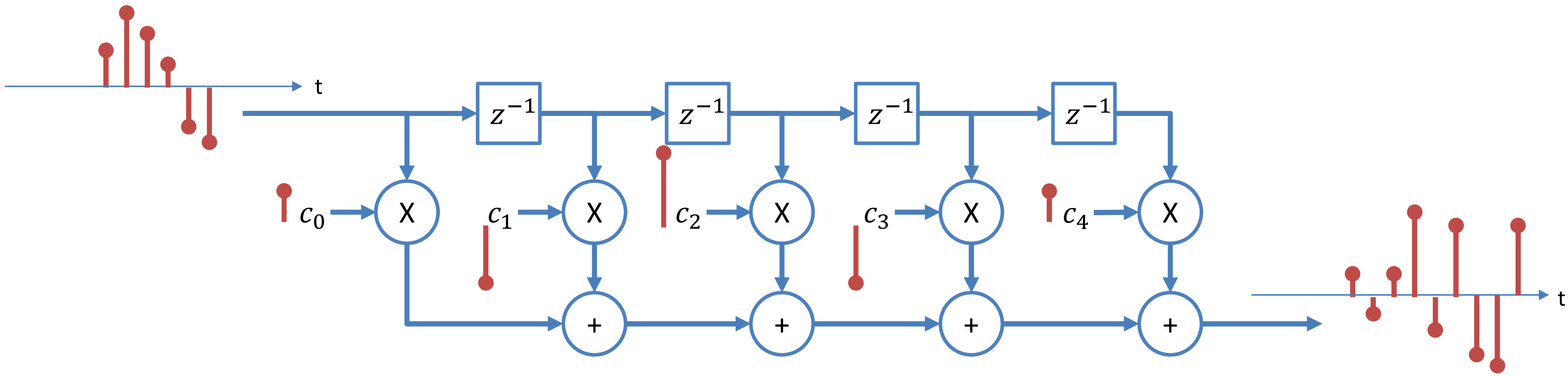
FIR Filters



The output is the **convolution** of the input with the coefficients...

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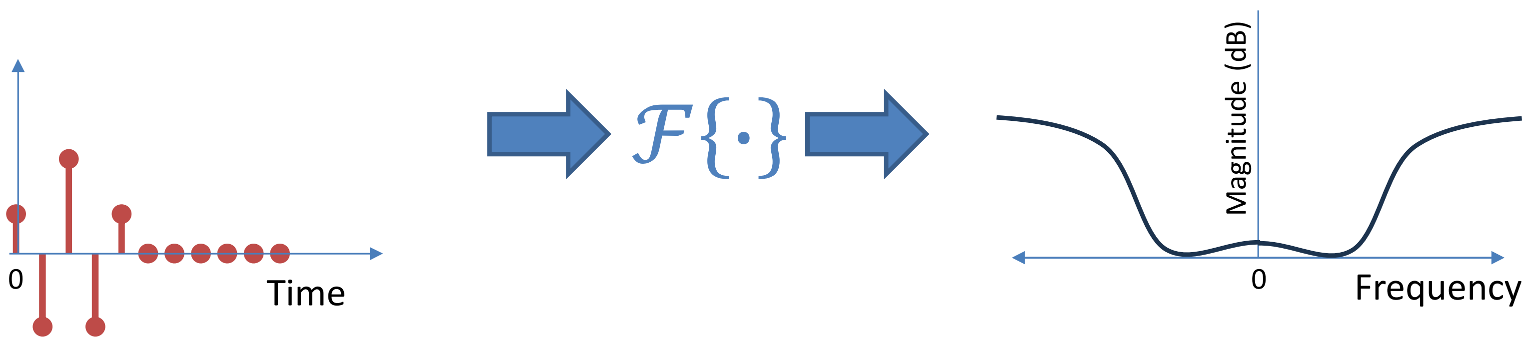

The Fourier Transform of the Impulse Response is the **Frequency Response**

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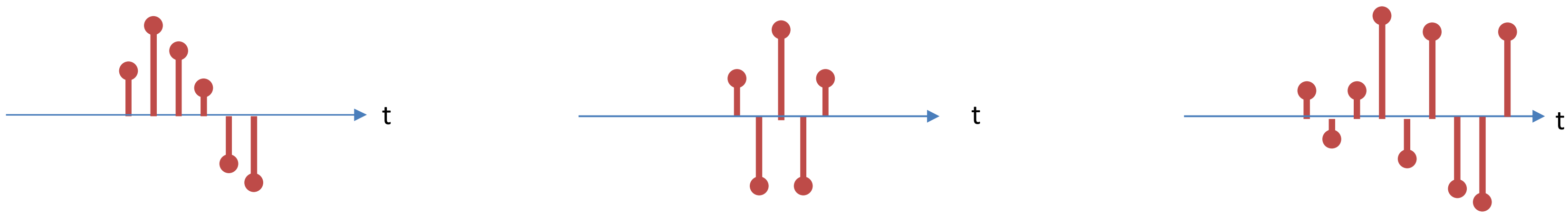
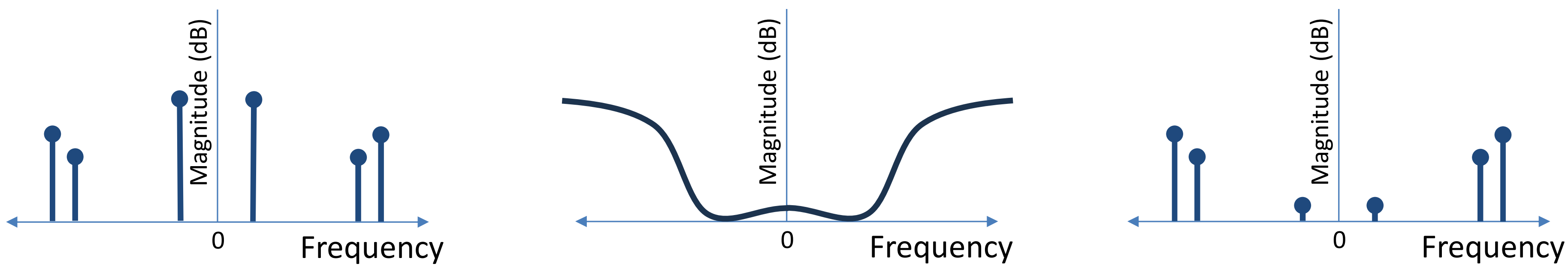
Convolution in the time domain …



… is **Multiplication** in the frequency domain



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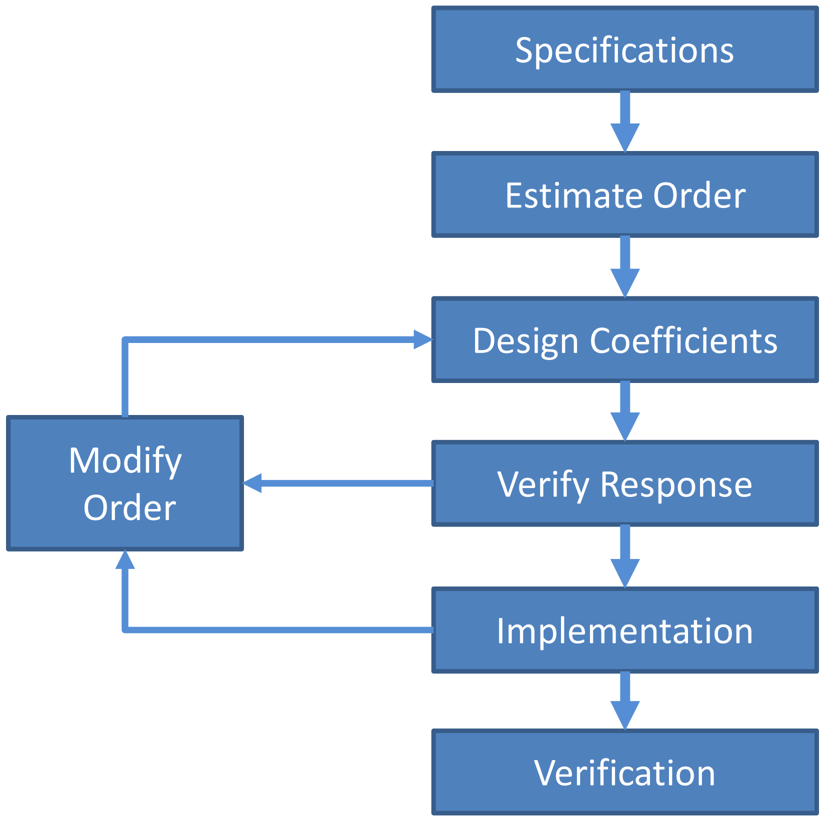
FIR Design Flow



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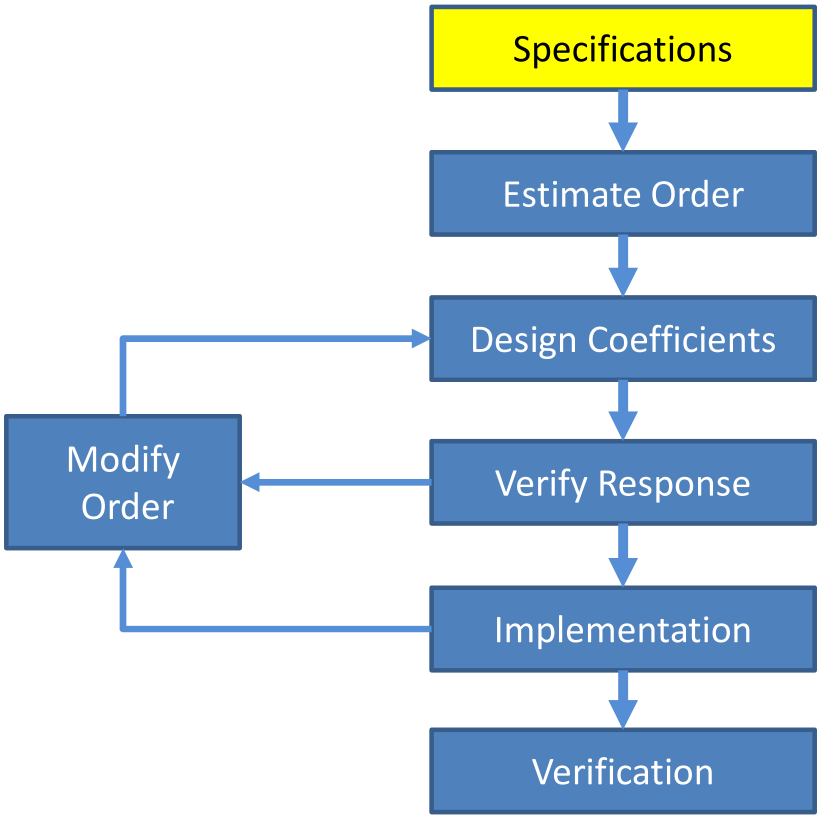
FIR Design Flow



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Filter Specification

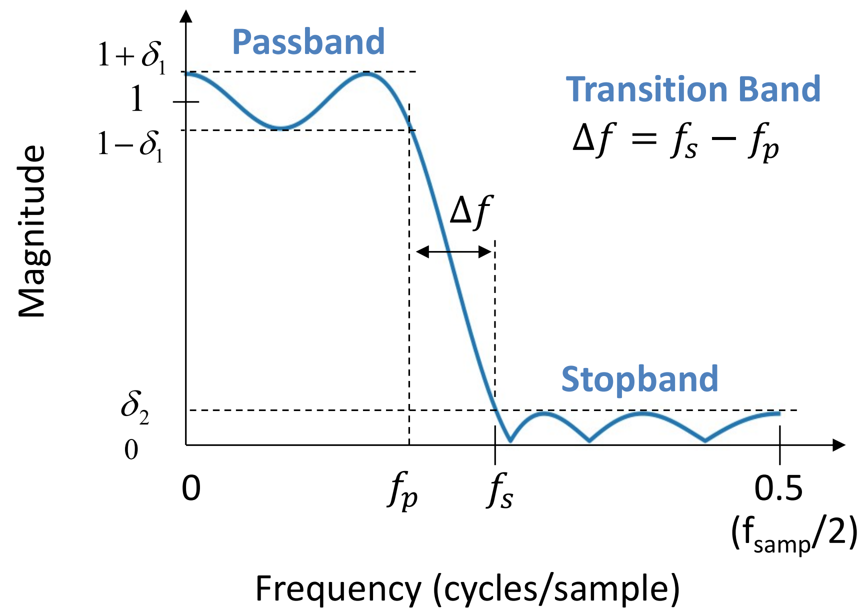
f_p: **Passband edge frequency** (cycles/sample)

δ_1: **Peak Passband Ripple**, in dB: $-20log_{10}(1-\delta_1)$

f_s: **Stopband edge frequency** (cycles/sample)

δ_2: **Peak Stopband Ripple**, in dB (rejection): $20log_{10}(\delta_2)$

Filter order: Total time delay or processing constraint



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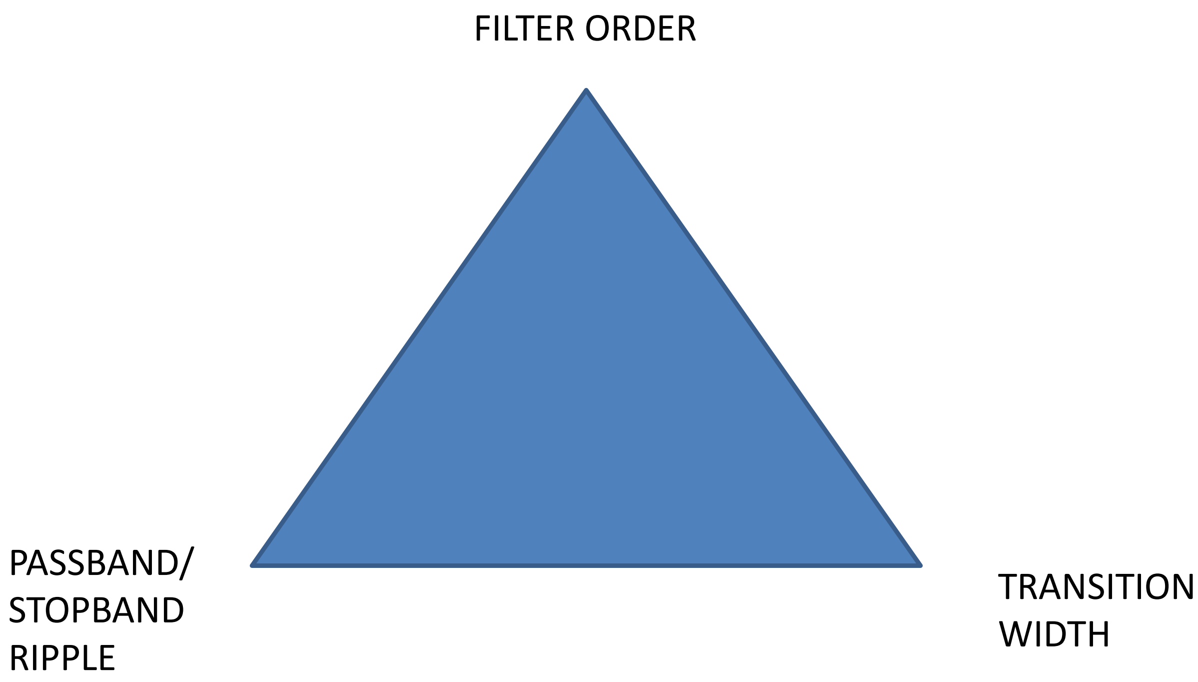
Trade Space



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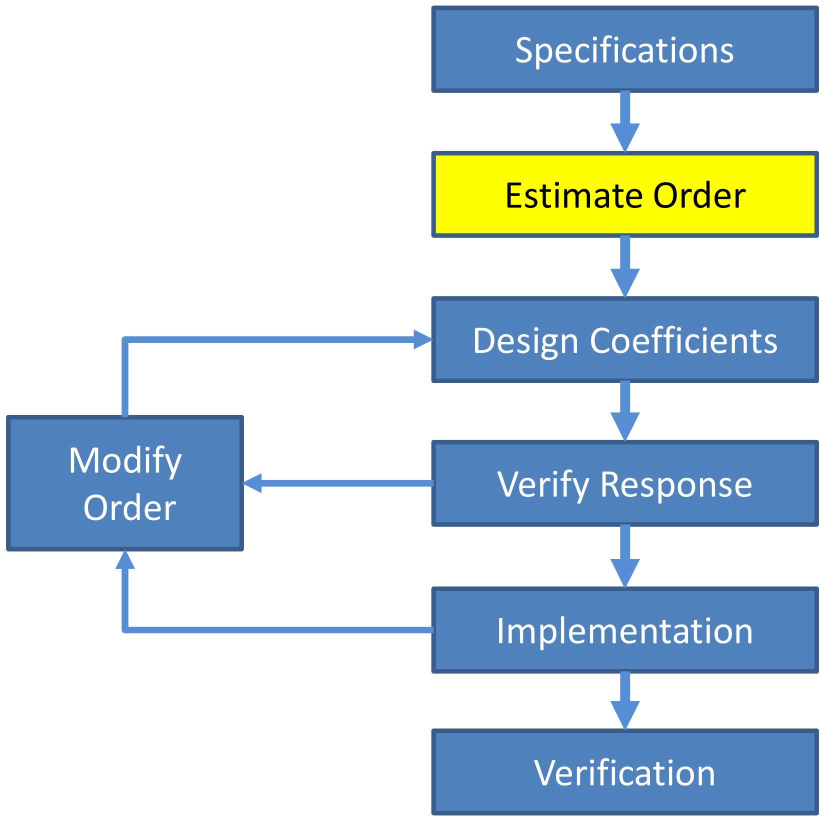
FIR Design Flow



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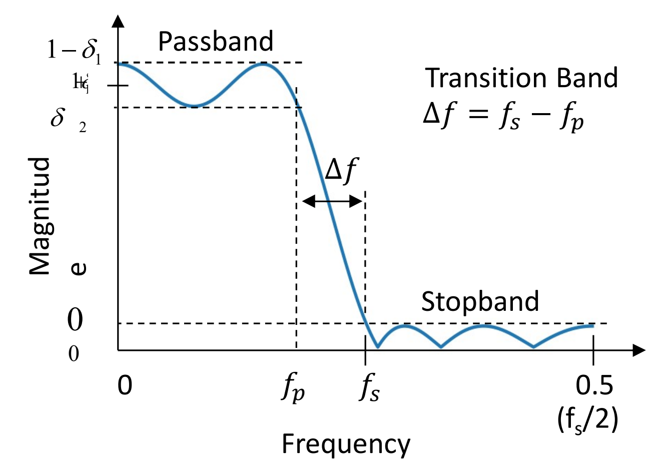
Estimate Number of Coefficients

`scipy.signal.kaiserord`

Kaiser's Estimators	
Parks-McClellan Filters	Kaiser Window Filters
$\dfrac{-10log_{10}(\delta_1\delta_2) - 13}{14.6(\Delta f)} + 1$	$\dfrac{A - 8}{14.4(\Delta f)} + 1$

fred harris Estimator
$\dfrac{A}{22(\Delta f)}$



Where A is attenuation in dB as a positive number:
$$A = -20log_{10}(\delta_2)$$

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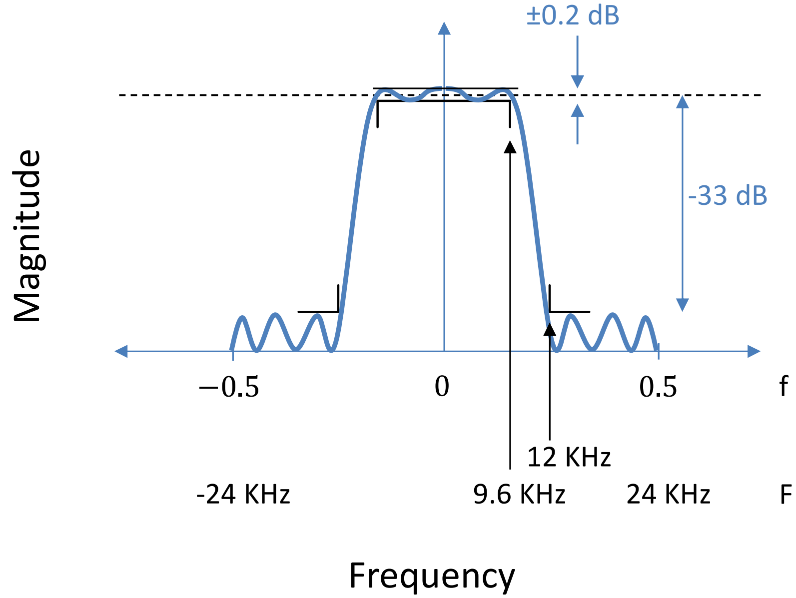
Example

<u>Low Pass Filter</u>
Passband ripple ±0.2 dB
Stopband rejection: -33 dB
Sampling rate: 48 KHz
Passband edge: 9.6 KHz
Stopband edge: 12 KHz



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Example

<u>Low Pass Filter</u>

Passband ripple ±0.2 dB

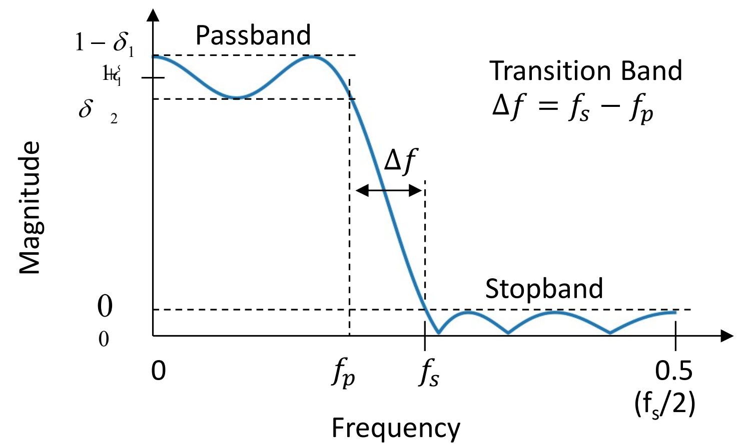
Stopband rejection: -33 dB

Sampling rate: 48 KHz

Passband edge: 9.6 KHz

Stopband edge: 12 KHz

$$\delta_1 = 10^{0.2/20} - 1 = 0.023$$
$$\delta_2 = 10^{-33/20} = 0.022$$
$$f_p = \frac{9600}{48000} = 0.2$$
$$f_s = \frac{12000}{48000} = 0.25$$



Kaiser: $$N \cong \frac{-10log_{10}(\delta_1\delta_2) - 13}{14.6(\Delta f)} + 1 = \frac{-10log_{10}(0.022 \cdot 0.023) - 13}{14.6 \cdot 0.05} = 28.3$$

fred harris: $$N \cong \frac{A}{22(\Delta f)} = \frac{33}{22 \cdot 0.05} = 30$$

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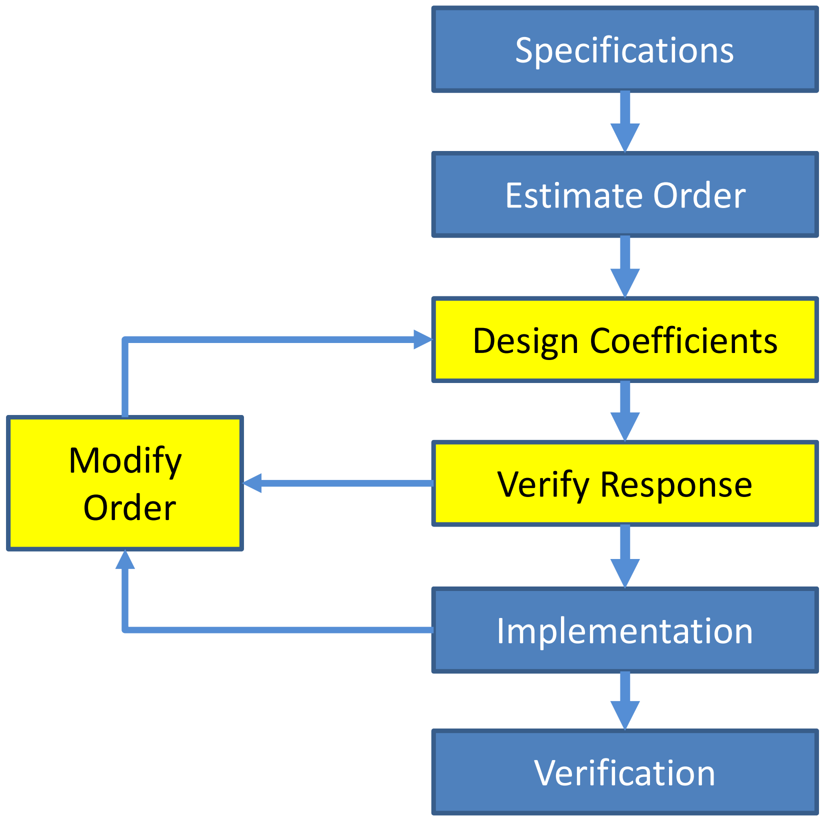
FIR Design Flow



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Direct FIR Realizations

Window method

Frequency Sampling

Equiripple (Parks-McClellan or Remez)

Least-Squares

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Direct FIR Realizations

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Truncate and Window

- Simple
- Non-optimal
- Typical windows
  - Bartlett, Modified Bartlett-Hanning (“Barthann”), Blackman, Blackman-harris, Bohman, Chebyshev, Cosine, DPSS, Flat Top, Gaussian, Hamming, Hann, Kaiser, Nutall, Parzen (de la Valle-Poussin), Boxcar, Taylor, Cosine-tapered (Tukey), Triangular
- All of these windows are available in Python!
- https://docs.scipy.org/doc/scipy/reference/signal.windows.html

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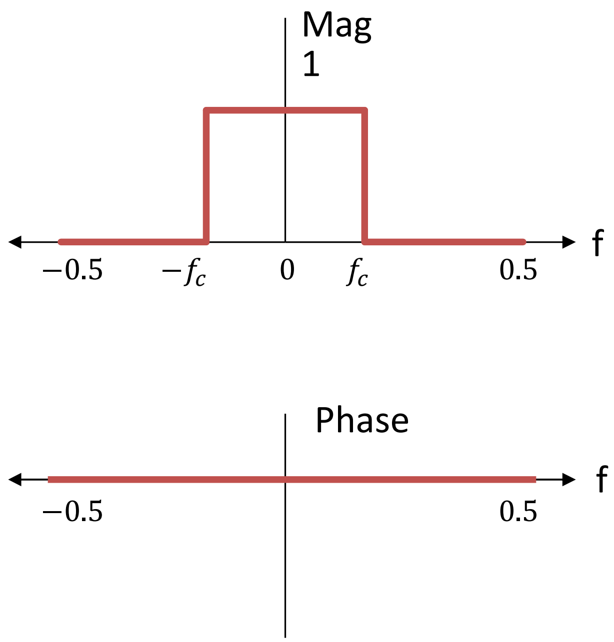
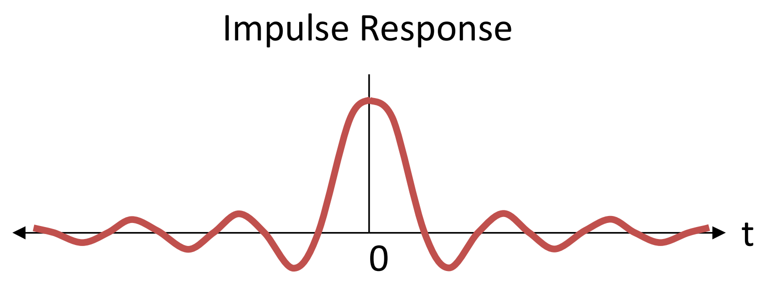
Ideal Lowpass Filter

$H(f) = 1, 0 \leq |f| \leq f_c$
$H(f) = 0, |f| > f_c$





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Ideal Lowpass Filter

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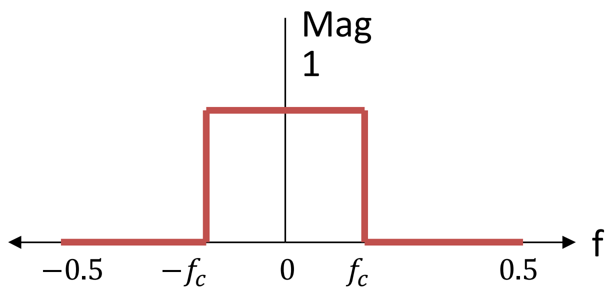

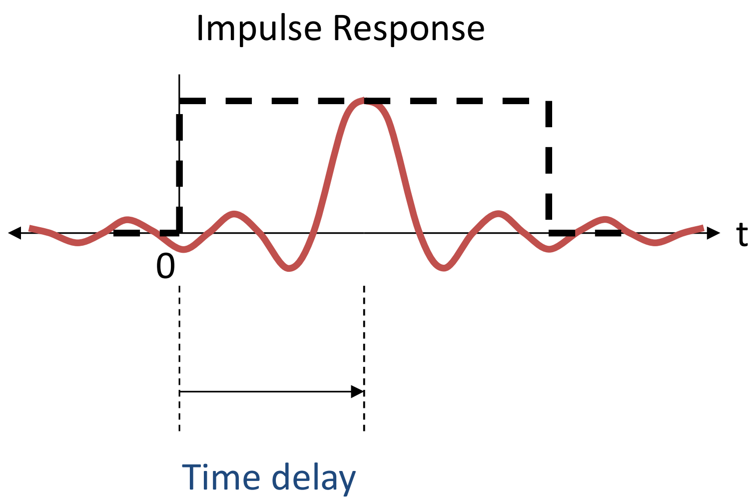

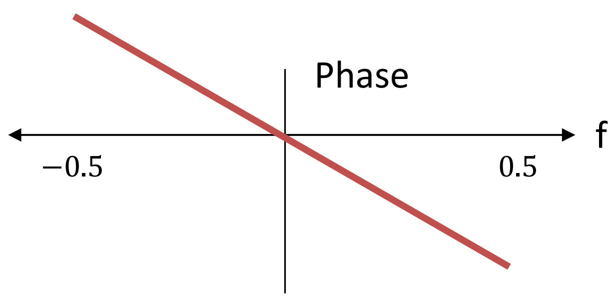

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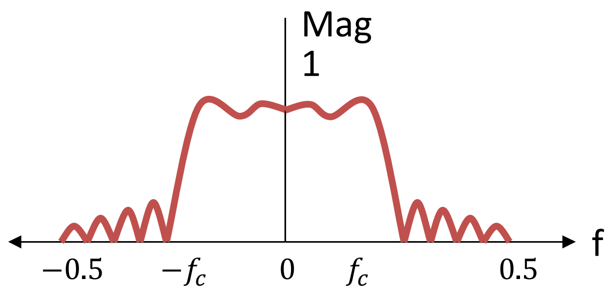
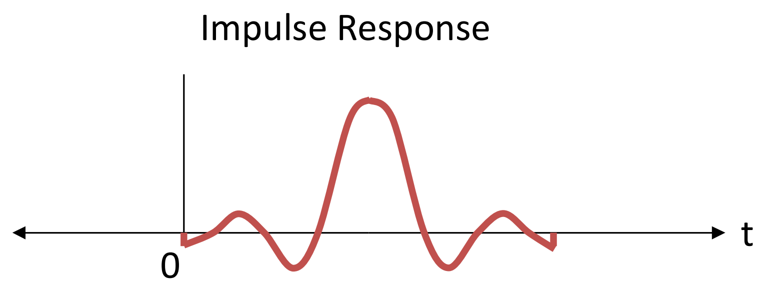
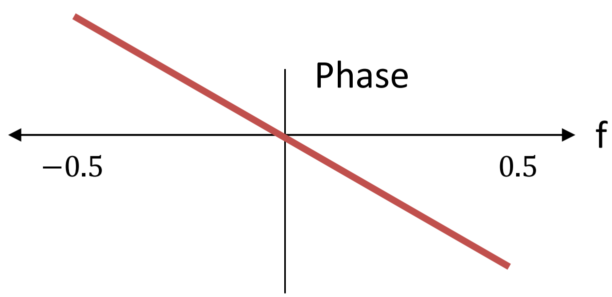
Rectangular Window

$H(f) = 1, 0 \leq |f| \leq f_c$
$H(f) = 0, |f| > f_c$







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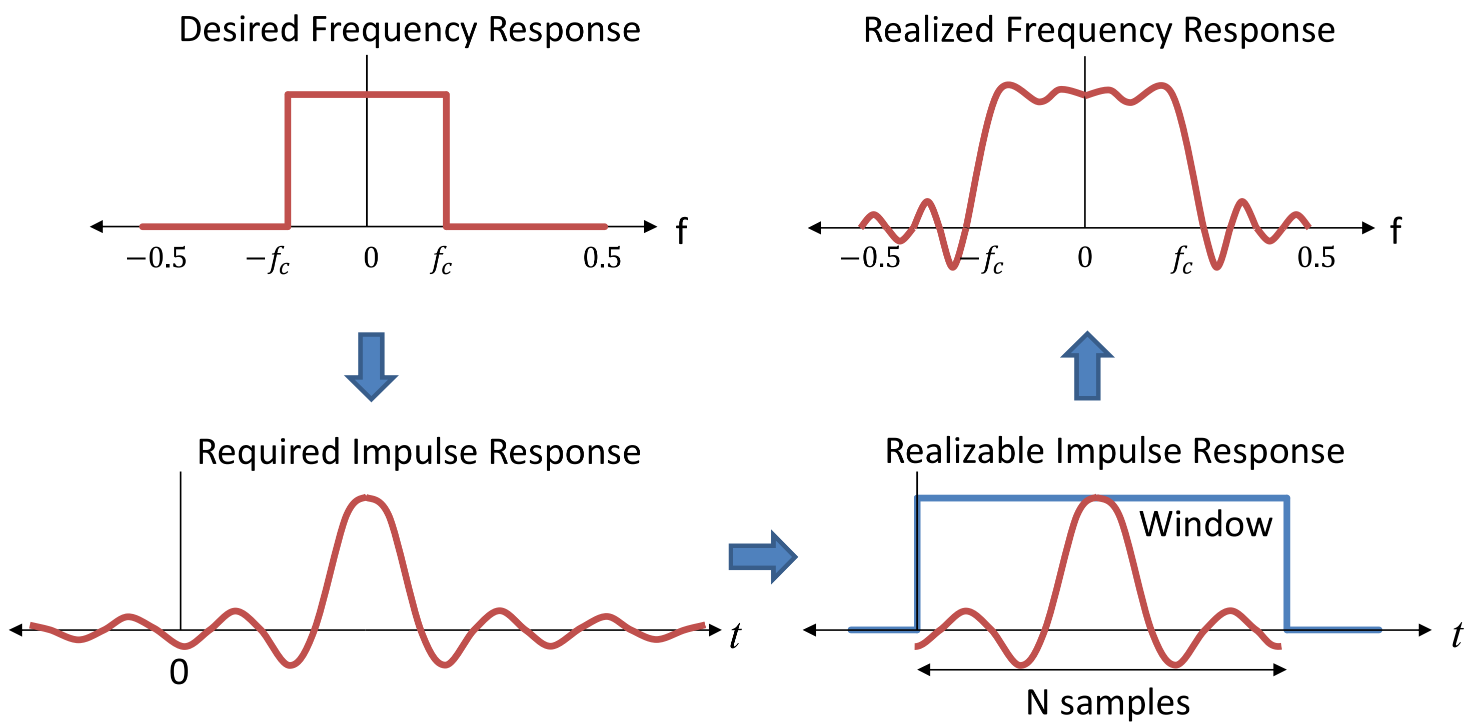
Rectangular Window



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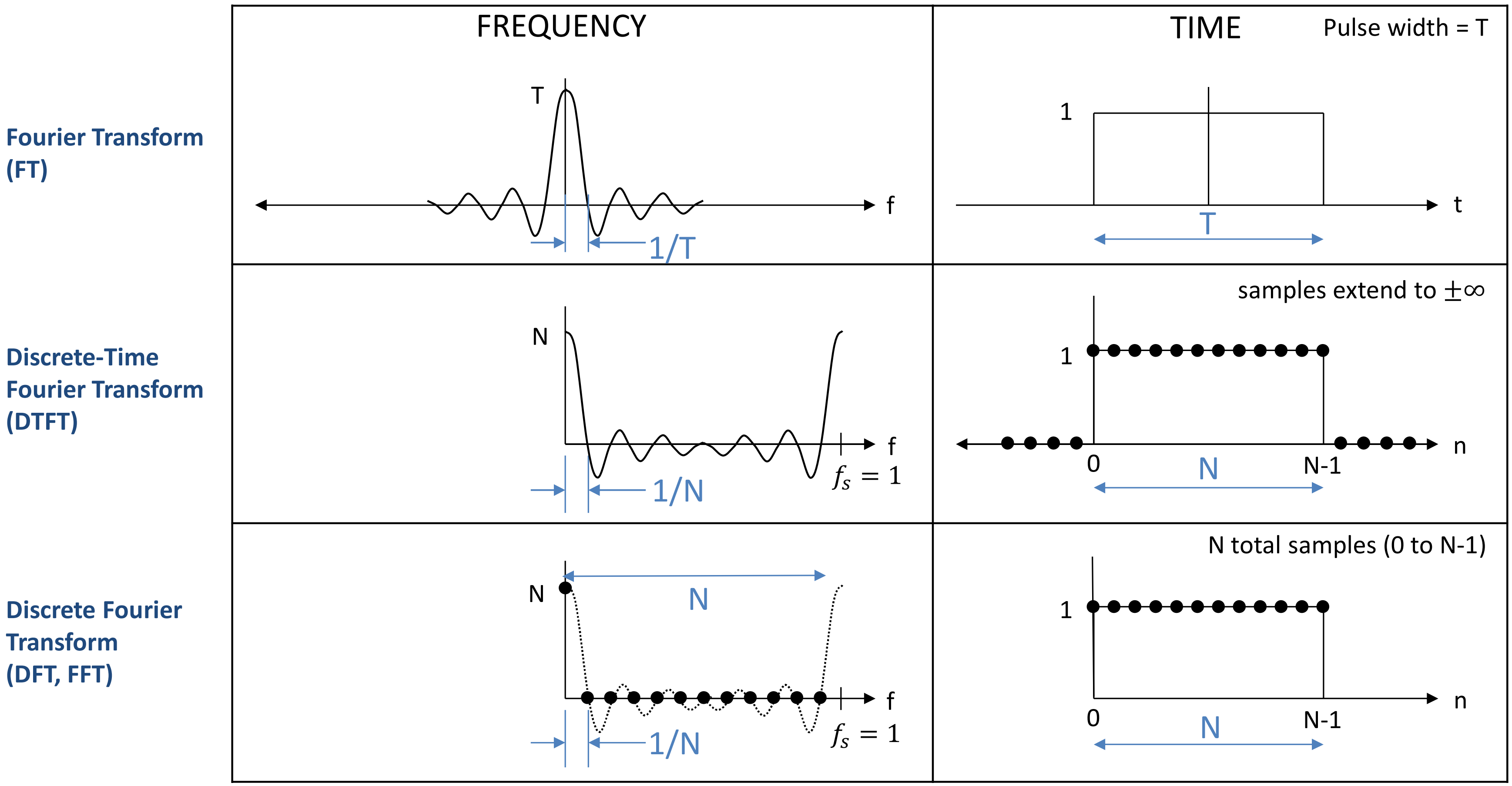
Transform Review



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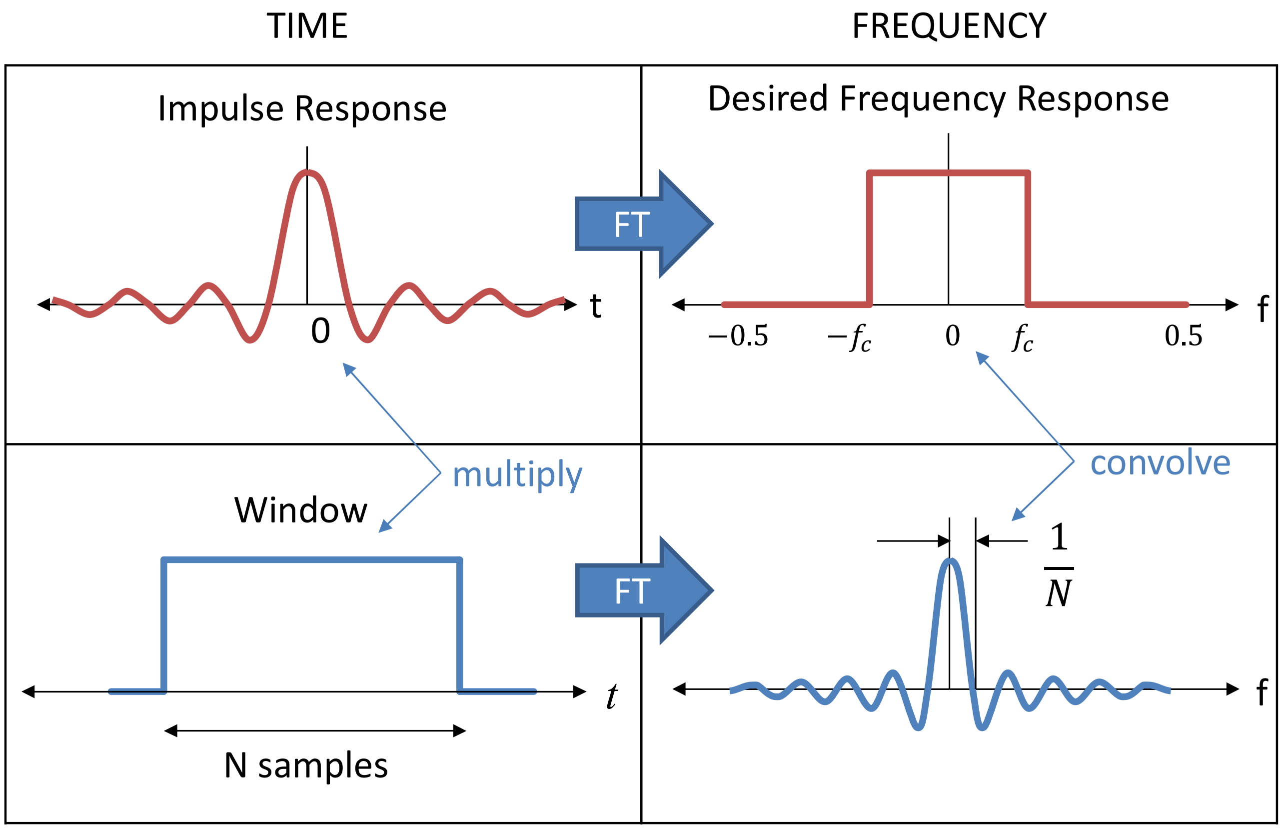
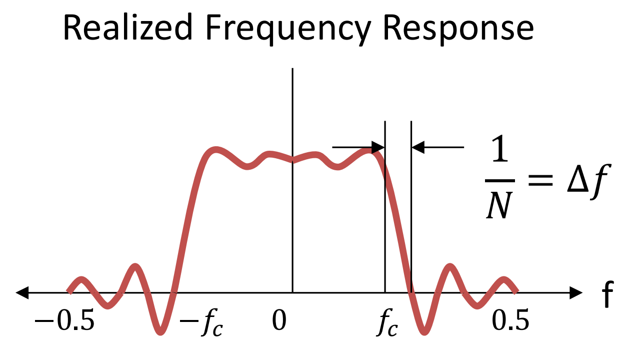
Rectangular Window





$$N = \frac{1}{\Delta f}$$

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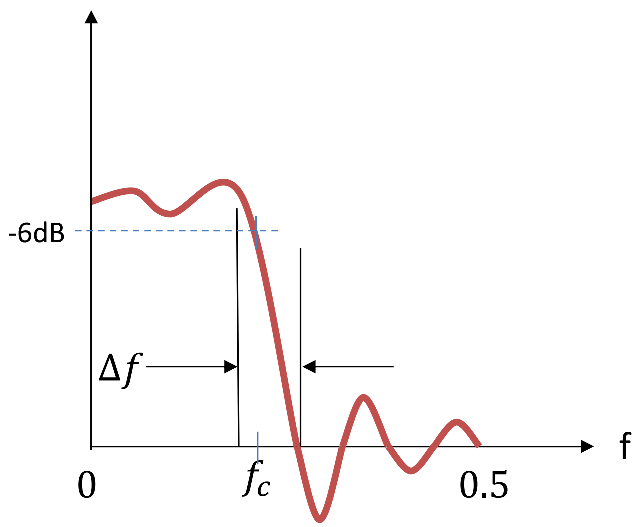
Rectangular Window

$N = \frac{1}{\Delta f}$ coefficients required to complete transition band



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Parks-McLellan Filters	Kaiser Window Filters
$\frac{-10log_{10}(\delta_1\delta_2)-13}{14.6(\Delta f)}+1$	$\frac{A-8}{14.4(\Delta f)}+1$

fred harris Estimator
$N \cong \frac{A}{22(\Delta f)}$

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Alternate Window

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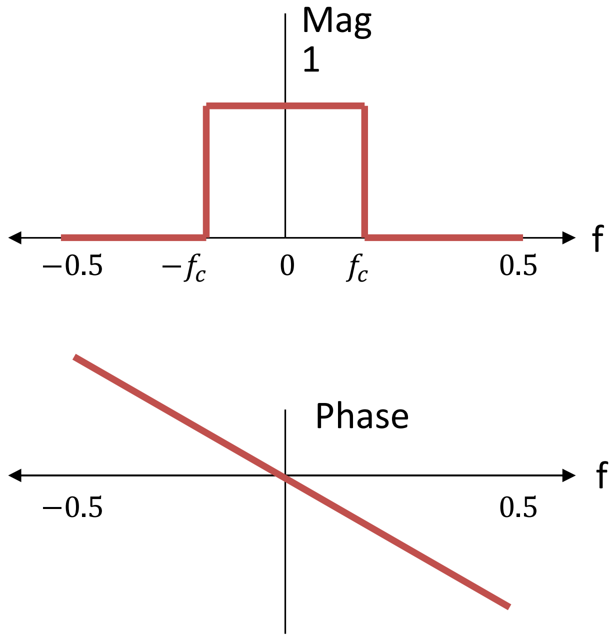

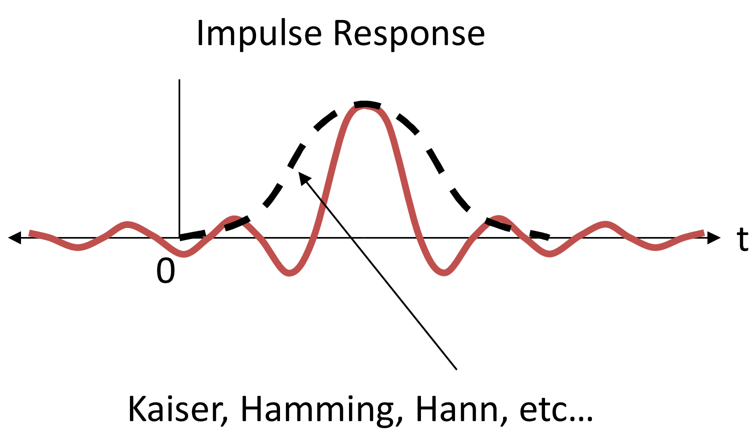

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Alternate Window

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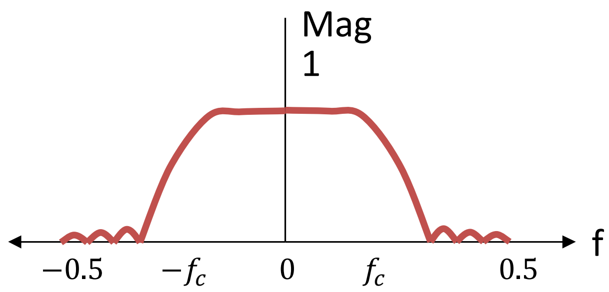

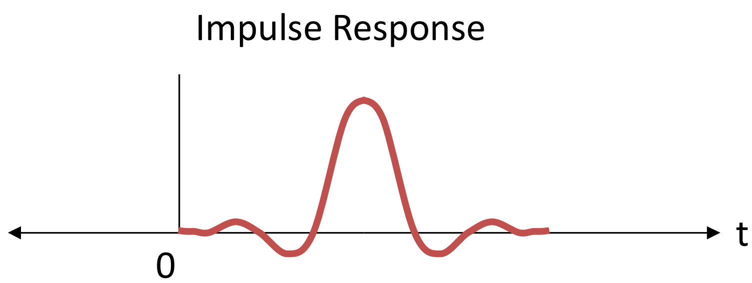

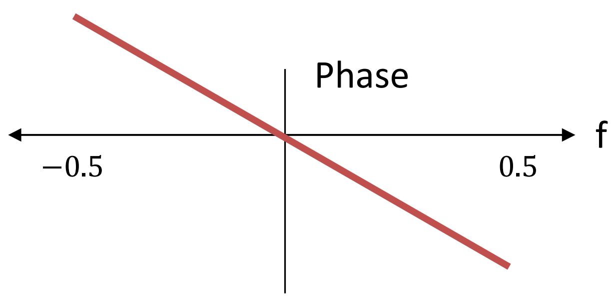

Reduced passband
and stopband ripple!
…But wider transition band
for a given length N

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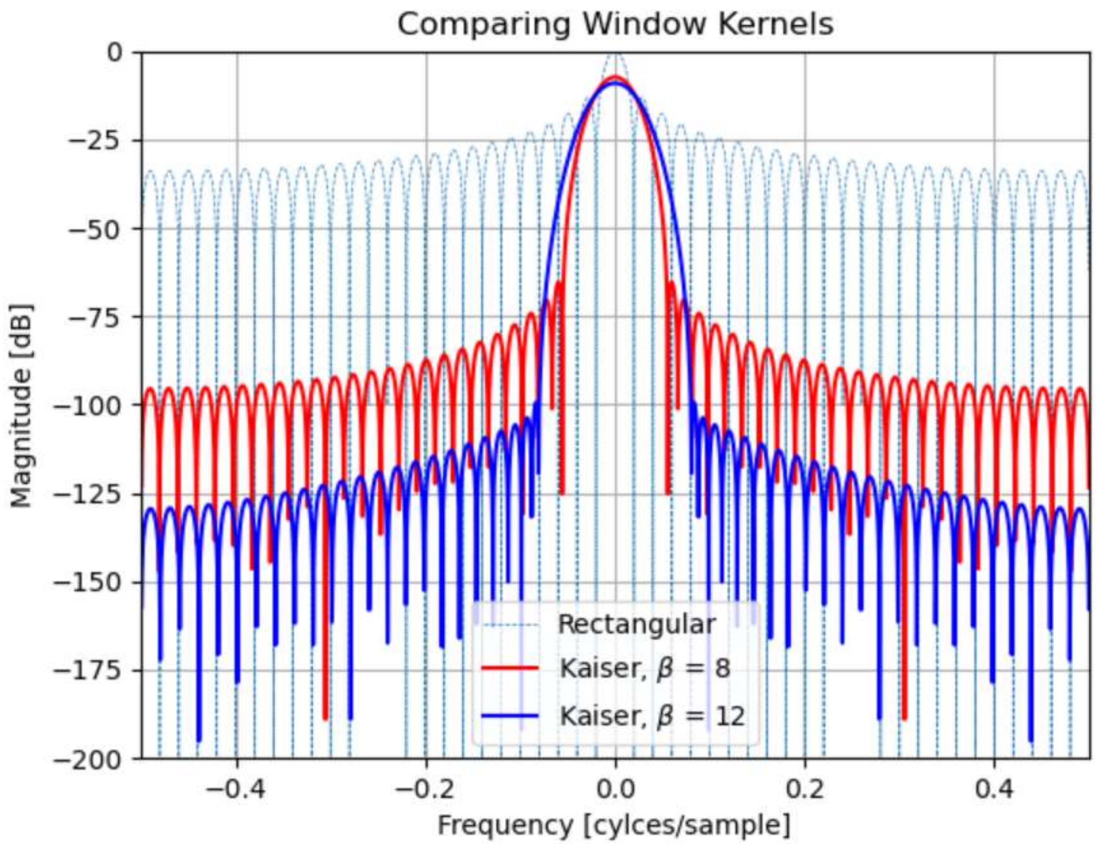
DTFT of the Kaiser Window



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Window Summary

Establish desired frequency response

Determine impulse response (using Inverse Fourier Transform)

Sample, truncate and window

Estimate number of taps and Kaiser beta:
`scipy.signal.kaiserord`

Create filter coefficients
`scipy.signal.firwin` OR
`scipy.signal.firwin2`

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Direct FIR Realizations

Window method

Frequency Sampling

Equiripple (Parks-McClellan or Remez)

Least-Squares

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Frequency Sampling

<u>Very</u> simple

Coefficients are the Inverse DFT of the sampled frequency response.

(In contrast to Inverse FT for Windowing method).

Sub-optimal

Longer filter length required over optimized approaches.

Will be exact at frequency sample locations.

Larger ripple in between.

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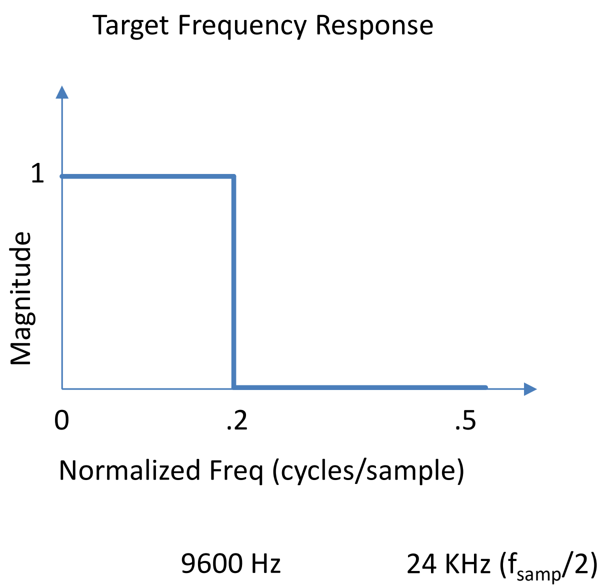
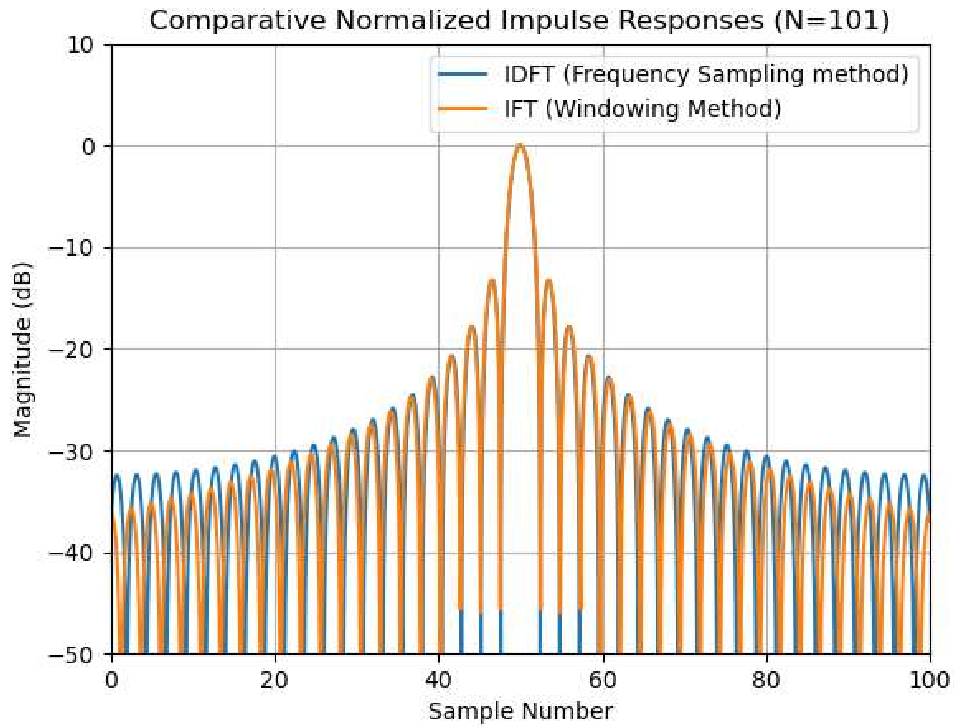
Frequency Sampling Example





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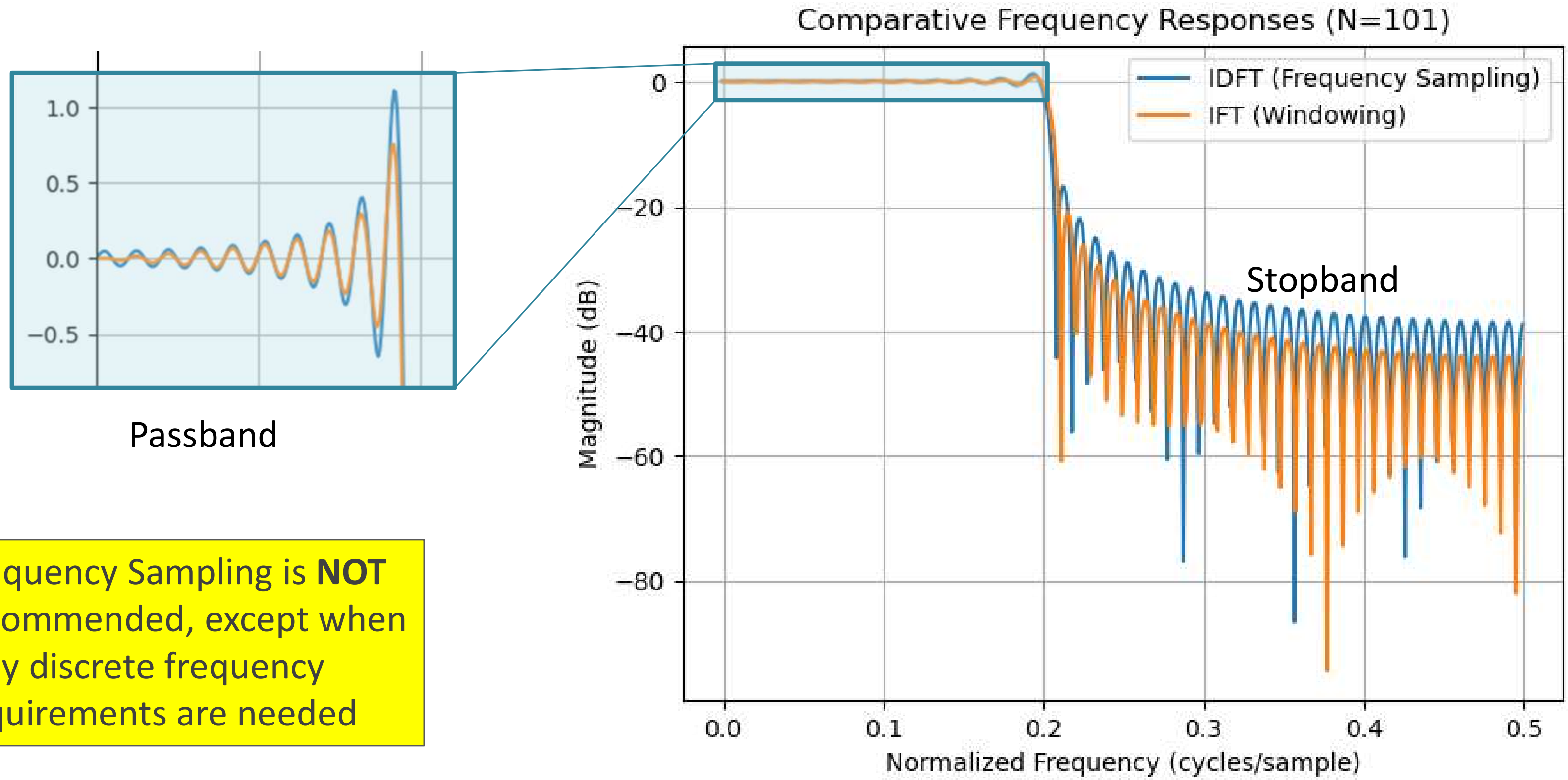
Frequency Sampling



Frequency Sampling is **NOT** recommended, except when only discrete frequency requirements are needed

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Direct FIR Realizations

Window method

Frequency Sampling

Equiripple (Parks-McClellan or Remez)

Least-Squares

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Optimized Algorithms

Equiripple (Parks-McClellan, Remez)

“best” in the mini-max sense: minimize the maximum error

Ideally suited when a specific tolerance must be met

Smallest peak error in passband and stopband

Least Squares

“best” in the least squares sense: minimize the least square error

Will have greater excursions from target than Equiripple, but lower rms error overall.

Smallest stopband energy and rms passband error

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Design Tools for Optimized Algorithms

	Python scipy.signal	MATLAB	Octave
Equiripple	remez	firpm	remez
Least Squares	firls	firls	firls

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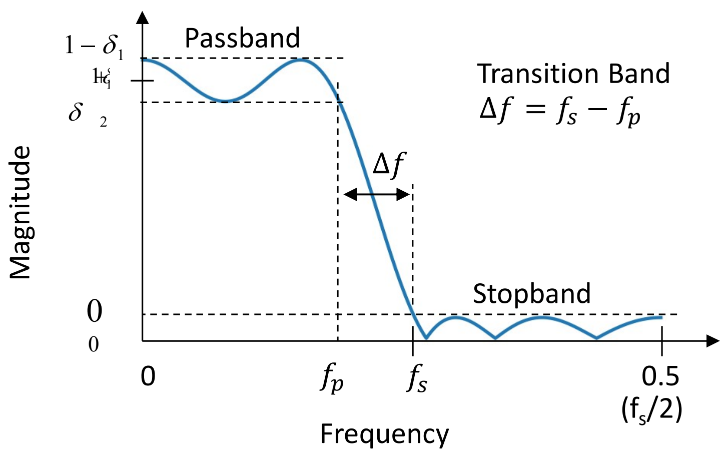
Design Tools for Optimized Algorithms




```
# bands (nyq = 1 for firls, nyq = 0.5 for remez)
ls_bands = [0, 2*pass_edge, 2*stop_edge, nyq]
remez_bands = [0, pass_edge, stop_edge, nyq]

ls_coeff = sig.firls(n_taps, ls_bands, [1,1,0,0])
remez_coeff = sig.remez(n_taps, remez_bands, [1, 0])
```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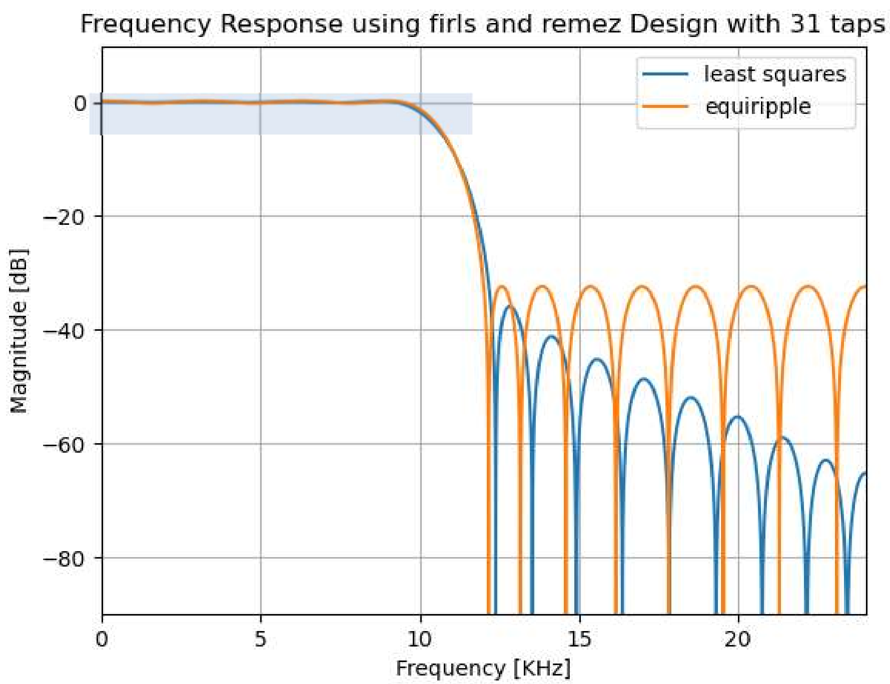
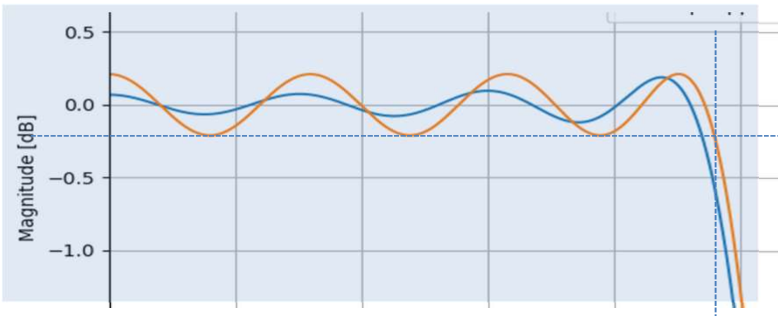
Example



Equiripple vs Least-Squared



Passband

9.6KHz

9.6 KHz Pass
12 KHz Stop
Fs=48 KHz
N = 31

```
ls_bands = [0, 9.6/2.4, 12/24, 1]
remez_bands = [0, 9.6/48, 12/48, 0.5]
```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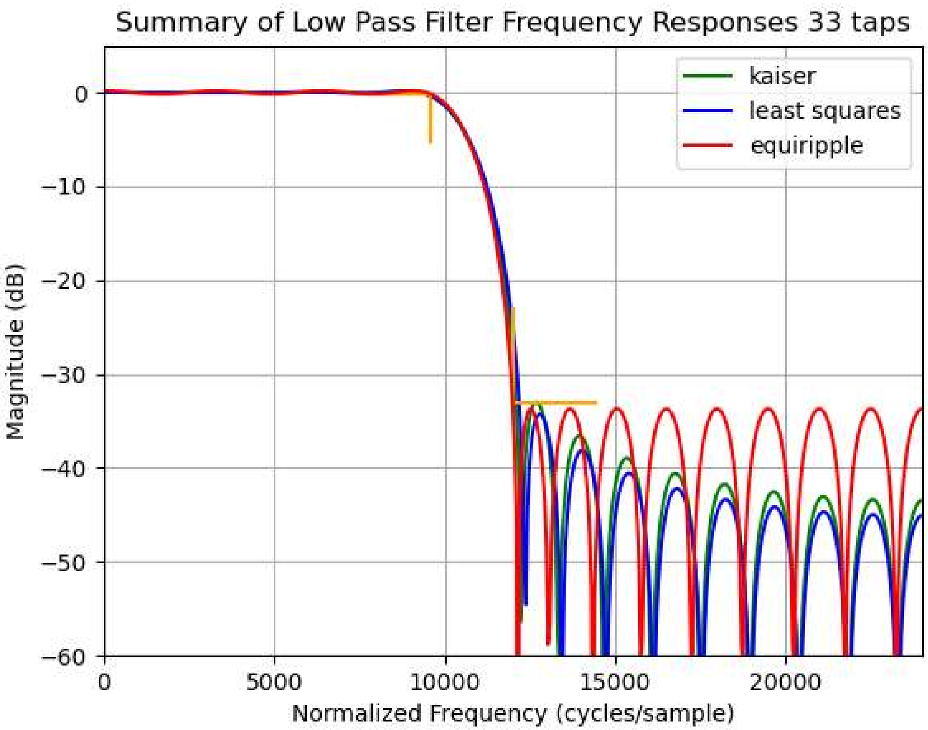
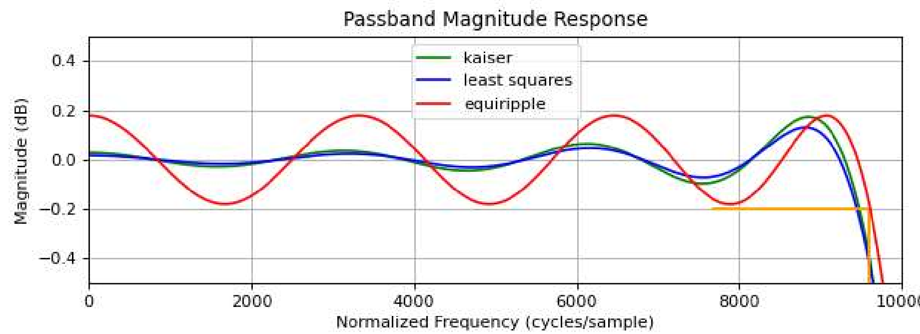
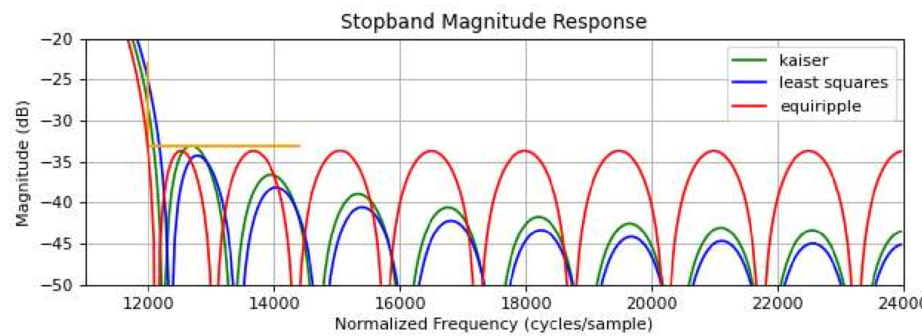
Performance Summary of Design Methods







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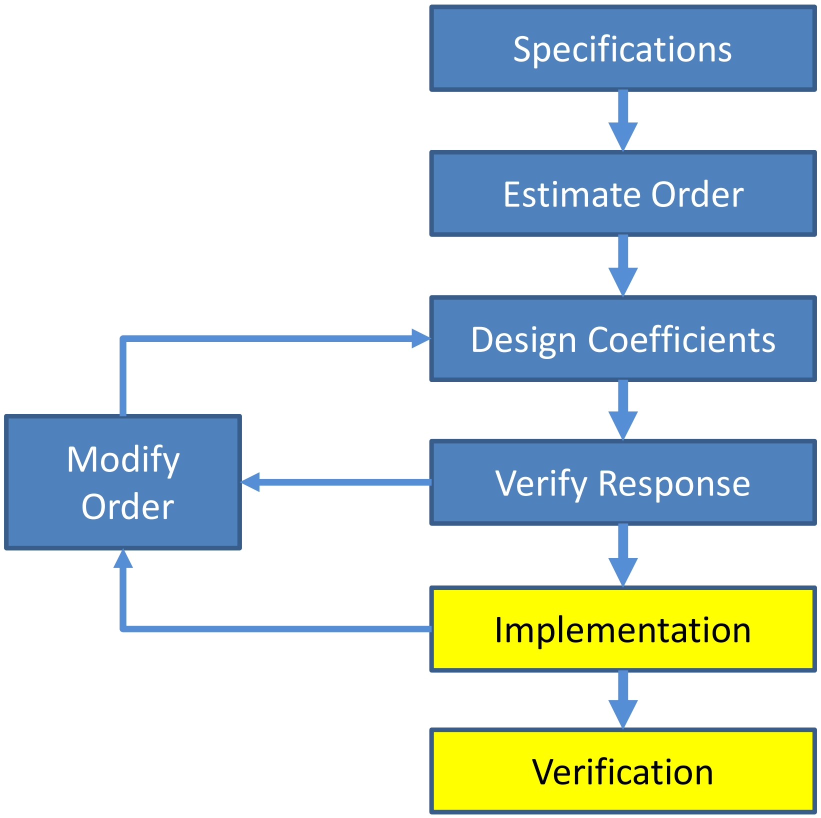
FIR Design Flow



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FIR Implementation and Verification

Further details specific to FIR Implementation and Verification and much more are covered in the upcoming course:

"**DSP For Wireless Communications**"

Starting October 10, 2024, through the Boston IEEE

IEEE

Sign-up Link: https://ieeeboston.org/courses/

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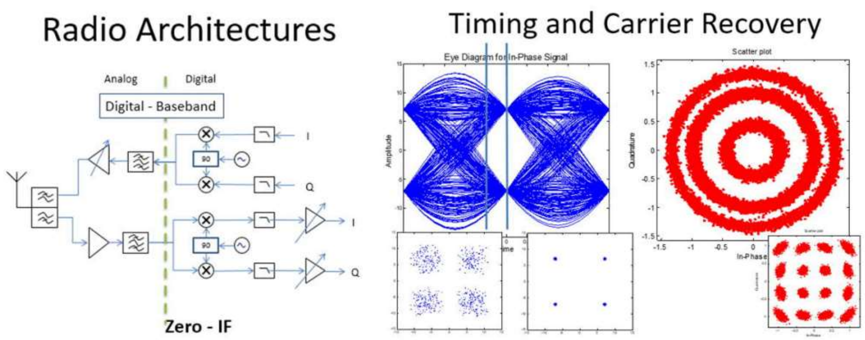
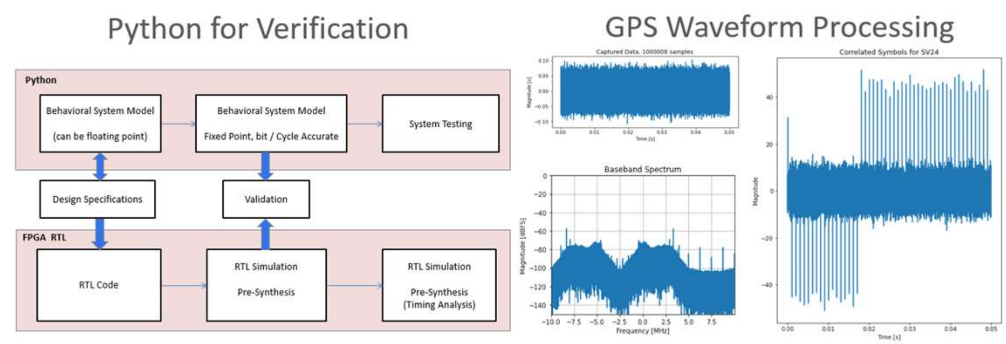
Want more DSP?





"DSP For Software Radio"

"Python Applications for Digital Design and Signal Processing"

Sign-up Link: https://dsprelated.com/courses

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Python Tools

SciPy

matplotlib

NumPy

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References

Alan V. Oppenheim, Alan S. Willsky, with S. Hamid. *Signals and Systems*, Prentice Hall

fred harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," in *Proceedings of the IEEE*, vol. 66, no. 1, pp. 51-83, Jan. 1978, doi: 10.1109/PROC.1978.10837.

Ronald W. Schafer, John R. Buck. *Discrete-Time Signal Processing*, Prentice-Hall Signal Processing Series by Alan V. Oppenheim

John G. Proakis, Dimitris K Manolakis. *Digital Signal Processing: Principles, Algorithms and Applications*, Prentice Hall

Paulo S. R. Diniz, Eduardo A.B. da Silva, Sergio L. Netto, *Digital Siganl Processing, System Analysis and Design*, Cambridge University Press

S.K. Mitra (1998). *Digital Signal Processing: A Computer-Based Approach,* McGraw-Hill, New York, NY

T.W. Parks and C.S. Burrus (1987). *Digital Filter Design,* New York: Wiley Interscience

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BACK-UP SLIDES

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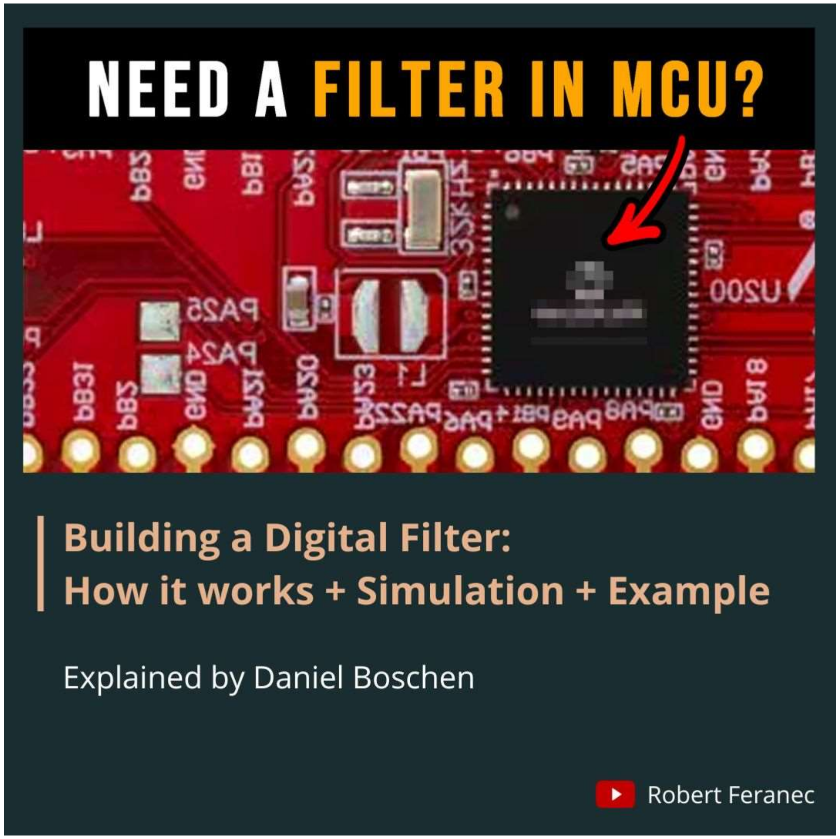
IIR Notch Filter on a SAM-E51 Processor

A very simple explanation and demo of digital filters.

https://www.youtube.com/watch?v=Aq_SOvR1Sxs&t=1s

(or Google "Robert Feranec Boschen Filter")



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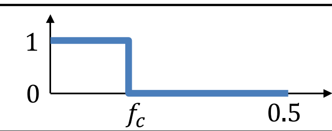
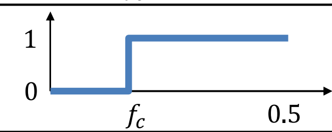
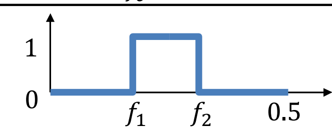
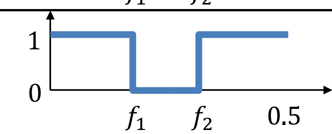
Impulse Responses for Ideal Filters

Filter Type	Impulse Response		Frequency Response $f_s = 1$
Lowpass, $h_{LP}[fc, n]$	$2f_c \mathrm{sinc}(2f_c n)$,	$-\infty \leq n \leq \infty$	
Highpass, $h_{HP}[fc, n]$	$h_{HP}[f_c, n] = h_{LP}[(0.5 - fc), n](-1)^n$	$-\infty \leq n \leq \infty$	
Bandpass, $h_{BP}[fc, n]$	$h_{LP}[f_2, n] - h_{LP}[f_1, n]$,	$-\infty \leq n \leq \infty$	
Bandstop, $h_{BS}[fc, n]$	$1 - 2(f_2 - f_1)$, $h_{LP}[f_1, n] - h_{LP}[f_2, n]$,	$n = 0$ $n \neq 0$	

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

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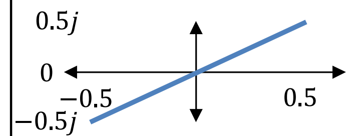
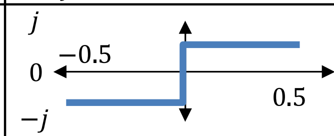
Impulse Responses for Ideal Filters

Filter Type	Impulse Response		Frequency Response $f_s = 1$
Differentiator, $h_d[n]$	$0,$ $\dfrac{(-1)^n}{n},$	$n = 0$ $n \neq 0$	
Hilbert, $h_H[n]$	$0,$ $\dfrac{1-(-1)^n}{\pi n},$	$n = 0$ $n \neq 0$	

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Estimate Beta and Order

Estimate of number of taps and beta using Kaiser Window:

$$\alpha_s = -20\log 10(\delta_2)$$

$$\begin{aligned}\beta &= 0.1102(\alpha_s - 8.7) \text{ for } \alpha_s > 50 \\ &= 0.5842(\alpha_s - 21)^{0.4} + 0.07886(\alpha_s - 21) \text{ for } 21 < \alpha_s \le 50 \\ &= 0 \text{ for } \alpha_s < 21\end{aligned}$$

$$N = \frac{\alpha_s - 8}{2.285\Delta\omega}$$

"Digital Signal Processing – A Computer Based Approach" – Sanjit K Mitra

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Using Parks-McClellan to Design Non-Linear Phase

From: https://dspguru.com/dsp/tricks/using-parks-mcclellan-to-design-non-linear-phase-fir-filter/

DSP Trick: Using Parks-McClellan to Design a Non-Linear Phase FIR Filter
From: ericj@primenet.com.nospam (Eric Jacobsen) Subject: DSP Trick - Using P-M to design a non-linear phase FIR filter. Date: 23 Oct 1999 00:00:00 GMT Newsgroups: comp.dsp THIS WORK IS PLACED IN THE PUBLIC DOMAIN
It is possible to use the Parks-McClellan algorithm to design FIR filters with non-linear phase response. For example, a FIR filter with the equivalent response of a Butterworth filter can be designed using the P-M routine.
First, take a common Butterworth description like that in Parks and Burrus where H(x) is a complex function. Create two PM input grids using the real and imaginary components of the Butterworth response. Use PM (or "remez" or whatever it's called on your favorite system) to design two FIR filters using the respective input grids, but turn the 'Hilbert' switch on for the one derived from the imaginary component. Sum the results, i.e., add together the nth coefficients of each filter to create a single N-tap filter.
The resulting FIR filter (assuming you've done your job to make sure everything converges) will have the desired response from the original expression used to generate the PM input grids.
Eric Jacobsen Minister of Algorithms

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Linear Phase FIR Types

Type	Length	Symmetry	Filter	Example
I	Odd	Symmetric	Any	[1 2 1]
II	Even	Symmetric	Not High	[1 2 2 1]
III	Odd	Anti-Symm	Band	[1 0 -1]
IV	Even	Anti-Symm	Not Low	[1 2 -2 -1]

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Title Banner for Jupyter