Quantifying Signal Quality

Practical Tools for High-Fidelity Waveform Analysis



Quantifying Signal Quality – Practical Tools for High-Fidelity Waveform Analysis

Content is derived from Signal Processing and Python courses taught by Dan Boschen

Copyright © 2025 C. Daniel Boschen. All Rights Reserved.

This presentation is protected by U.S. and international copyright laws. No part of this presentation may be reproduced, distributed, or transmitted in any form or by any means, without the prior written permission of the author.

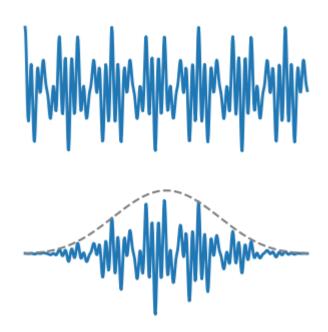
While every precaution has been taken in the preparation of this presentation, the author, publisher, and distribution partners assume no responsibility for any errors or omissions, or any damages resulting from the use of any information contained herein.

For additional resources and courses, visit dsp-coach.com

Intro / Goals

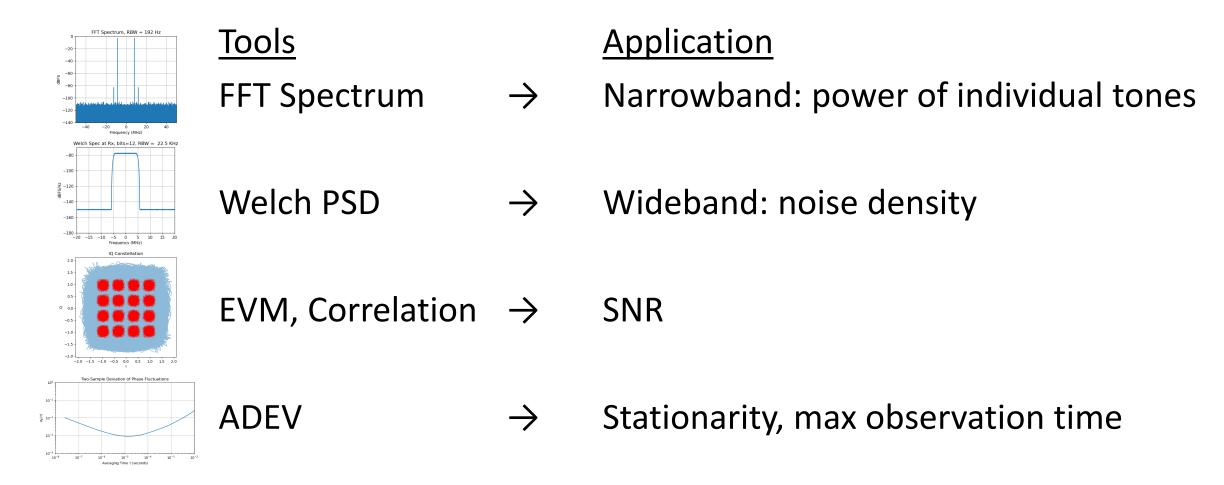
Accurate Signal and Noise Measurement

Time and Frequency Techniques



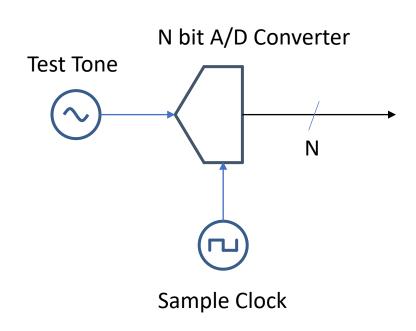
Post Processing Applications: Simplicity over Efficiency

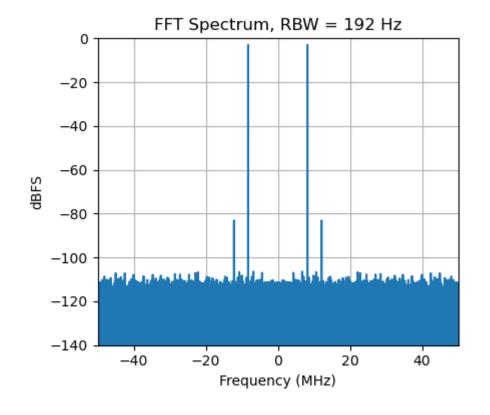
Summary

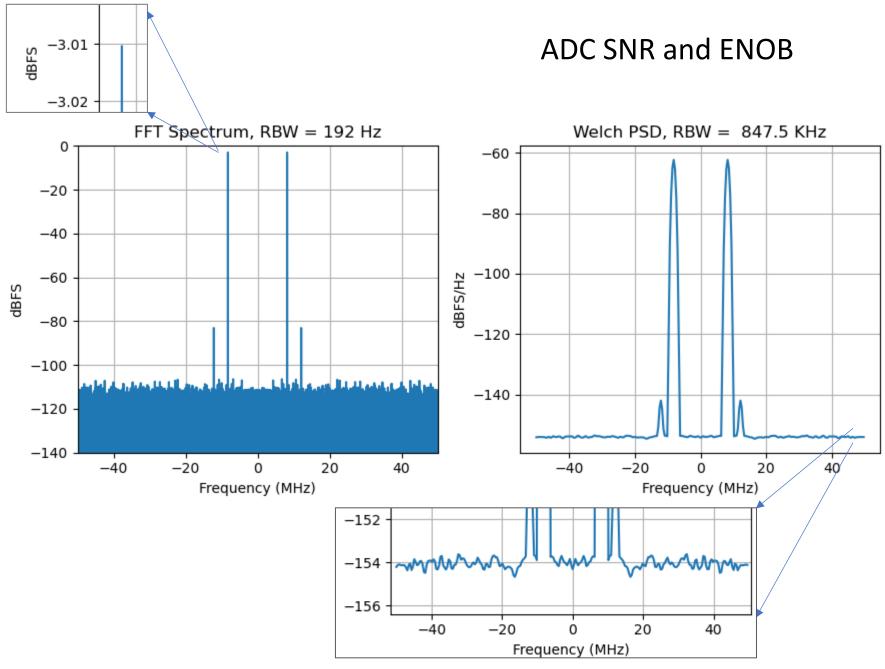


Compelling Examples

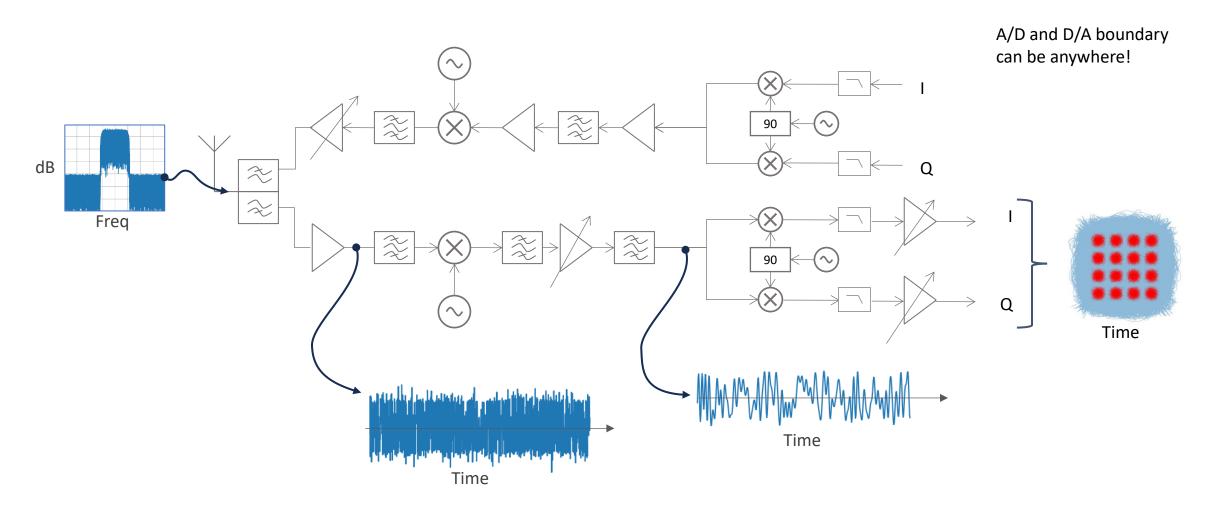
ADC SNR, Spurs and ENOB

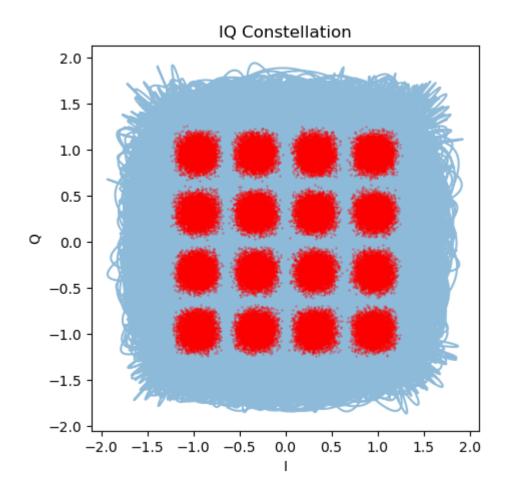






Waveform Quality in a Processing Chain

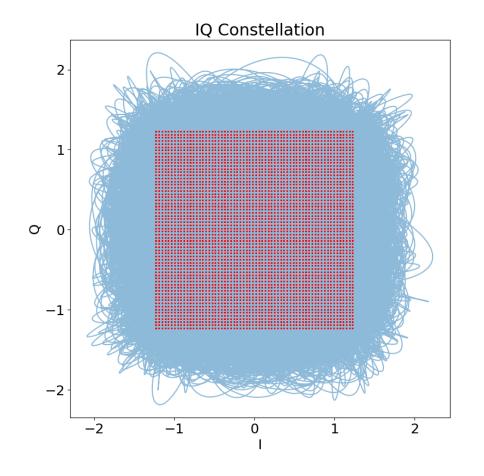


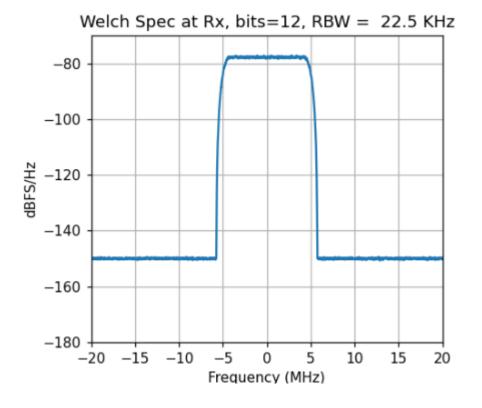


QAM 16 Constellation Showing Gray Code Mapping 0001 1001 1101 0101 1.0 0.5 0011 1011 1111 0111 O.0 0010 1010 1110 0110 -0.5 0000 1000 1100 0100 -1.0-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0

16-QAM

EVM, SNR, Spectrums

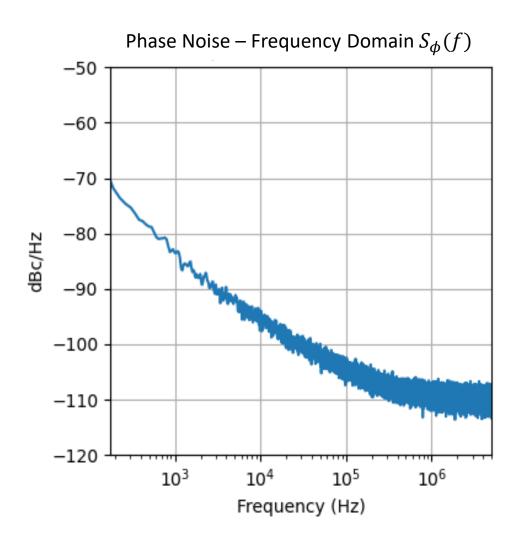


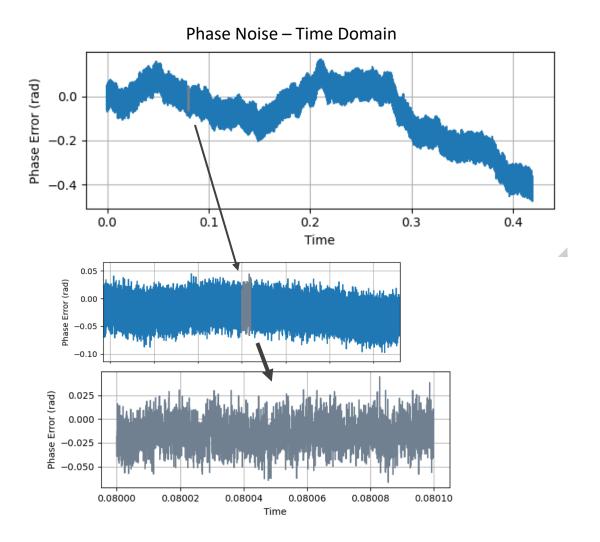


4096-QAM

EVM, SNR, Spectrums

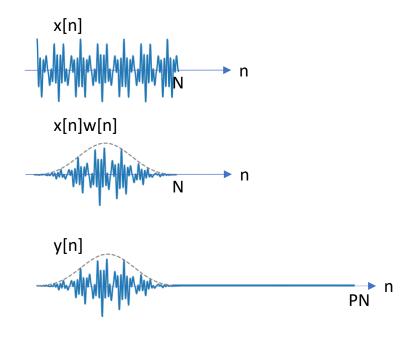
Consideration for Non-Stationary Signals





Power Measurement of Tones

FFT Spectrum Processing (for Tones)



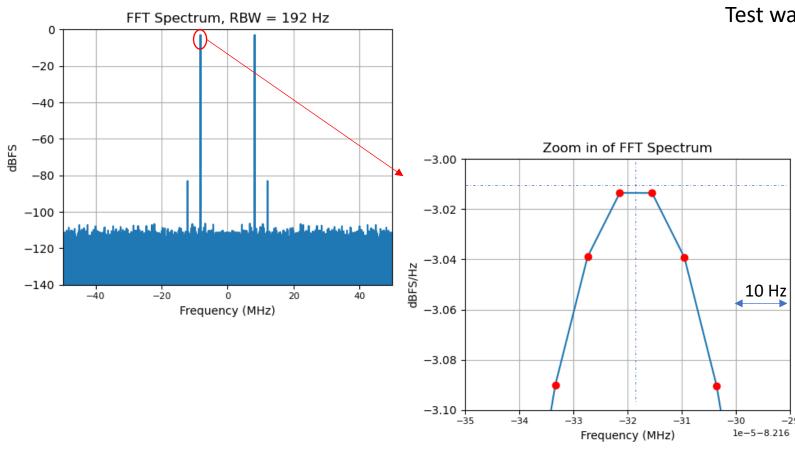
Duration N sets resolution bandwidth (RBW)

Window (Kaiser, vary β for dynamic range needed)

Zero pad (P=4 to 10x, next pow 2)

 $X[k] = \frac{1}{\sum w[n]} \sum_{n=0}^{N-1} y[n] W_{PN}^{nk}$ Scale FFT by sum of window

Results of FFT Spectrum



Test waveform:

 $x(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$

Sampling Rate: $f_s = 100 \text{ MHz}$

Total number of samples: N=1,048,576 Primary tone: f_1 =8.2163185 MHz, A_1 =1

Spur: f_2 =12.13 MHz, A_2 = 1e-4

Sampled and quantized to 12 bits to be x[n]

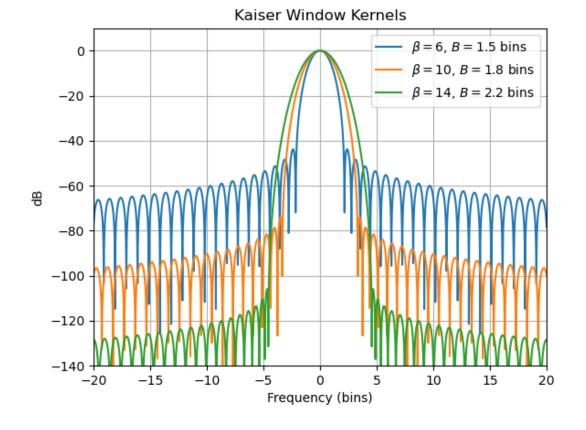
Resolution BW and Dynamic Range

Resolution Bandwidth and Dynamic Range: trade space with window used.

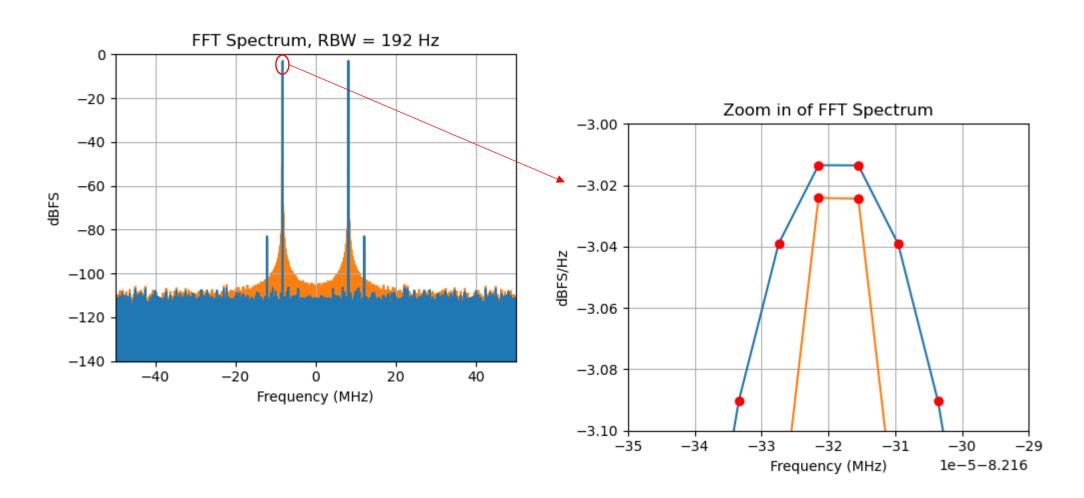
$$B = N \frac{\sum (w[n]^2)}{(\sum w[n])^2}$$

B result is in number bins of the original sequence, NOT the zero-padded length

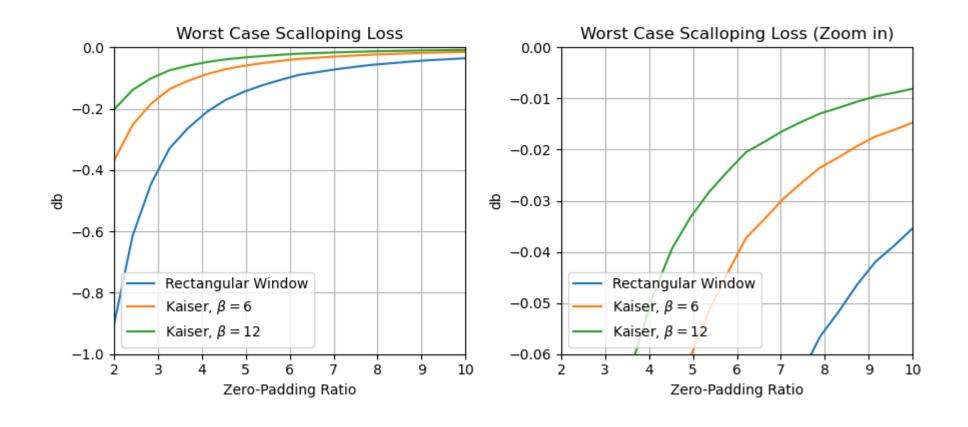
Resolution BW in Hz = $B \frac{f_s}{N}$



FFT Spectrum if not Windowed

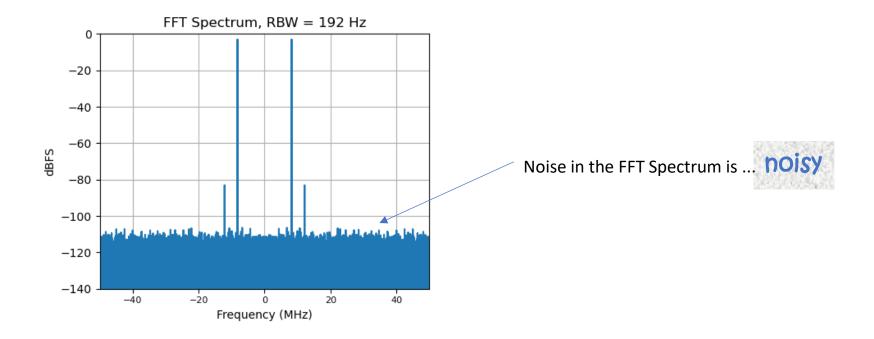


Scalloping Loss

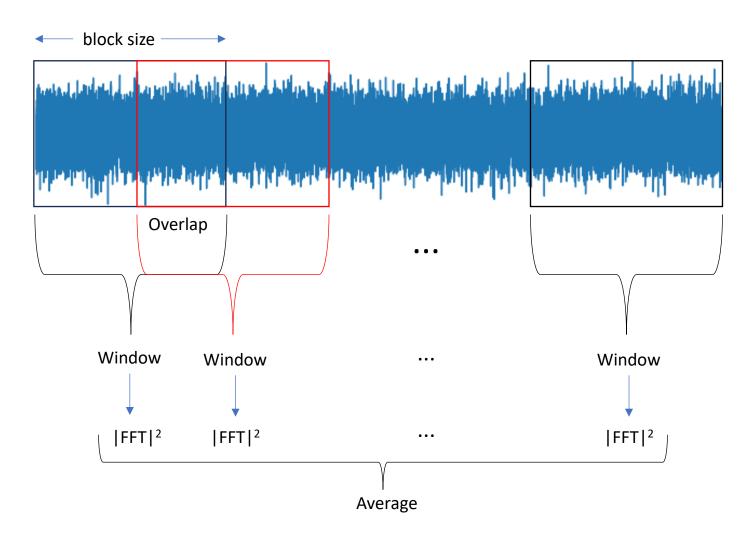


Power Measurement of Noise

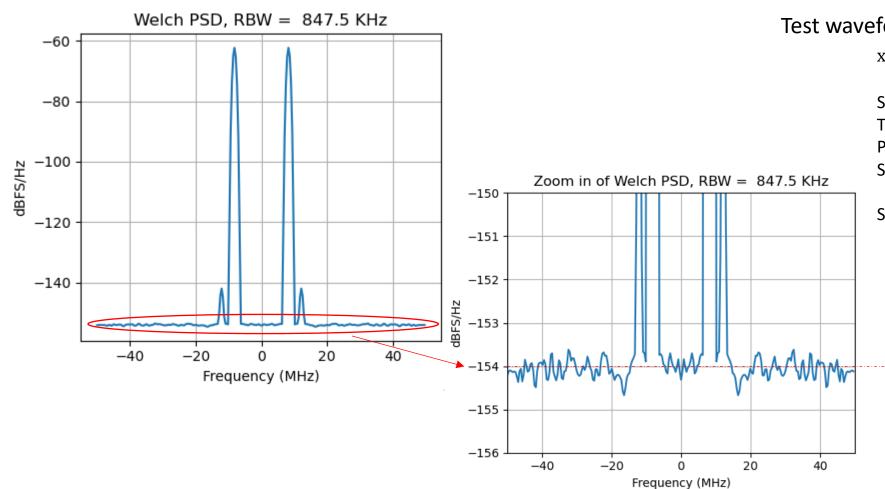
Motivation for using Welch



Welch Power Spectral Density for Noise



Results of Welch Power Spectral Density



Test waveform:

 $x(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$

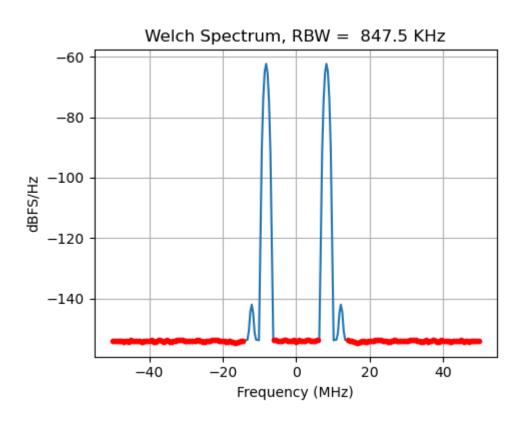
Sampling Rate: $f_s = 100 \text{ MHz}$

Total number of samples: N=1,048,576 Primary tone: f_1 =8.2163185 MHz, A_1 =1

Spur: f_2 =12.13 MHz, A_2 = 1e-4

Sampled and quantized to 12 bits to be x[n]

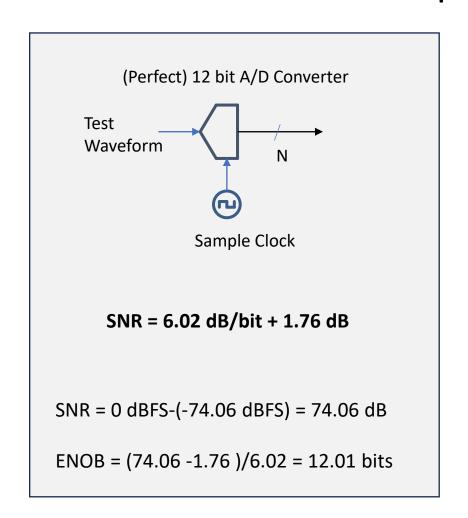
Results of Power Spectral Density



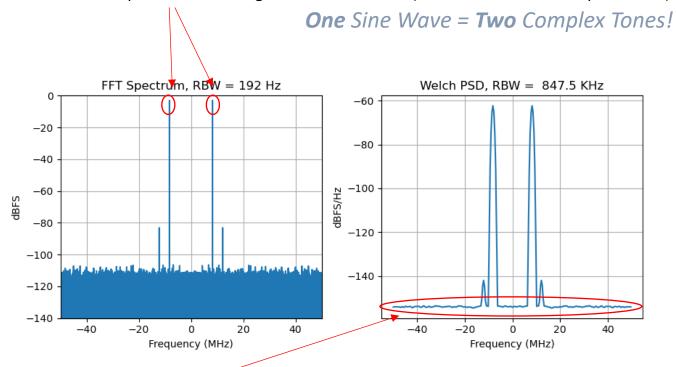
Extract the peaks and average the noise:

Result: -154.06 dBFS/Hz

ADC ENOB Example:



From FFT Spectrum: Total Signal Power = 0 dBFS (-3.01 dBFS as two complex tones)



From Welch PSD:

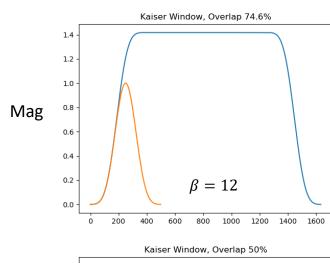
Total noise power -154.06 dBFS/Hz spread over 100 MHz

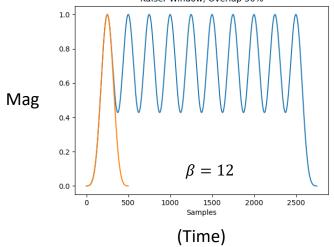
 $= -154.06 \text{ dBFS/Hz} + 10\log_{10}(100e6)$

= -74.06 dBFS

Recommended Overlap for Windows

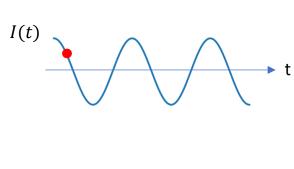
Window	Optimum Overlap
Hann	0.5
Kaiser, $\beta = 6$	0.637
Kaiser, $\beta=8$	0.692
Kaiser, $eta=10$	0.735
Kaiser, $\beta=12$	0.746
Kaiser, $eta=14$	0.768
Kaiser, $\beta=16$	0.772

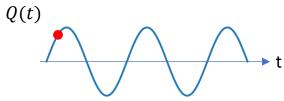


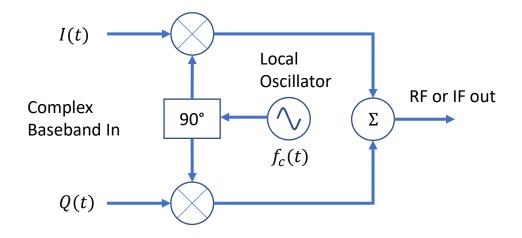


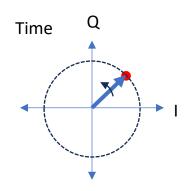
SNR using EVM and Correlation

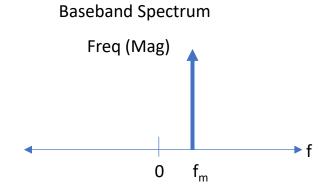
Brief "IQ" Intro

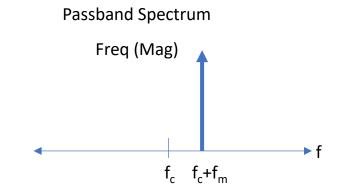












Correlation Coefficient

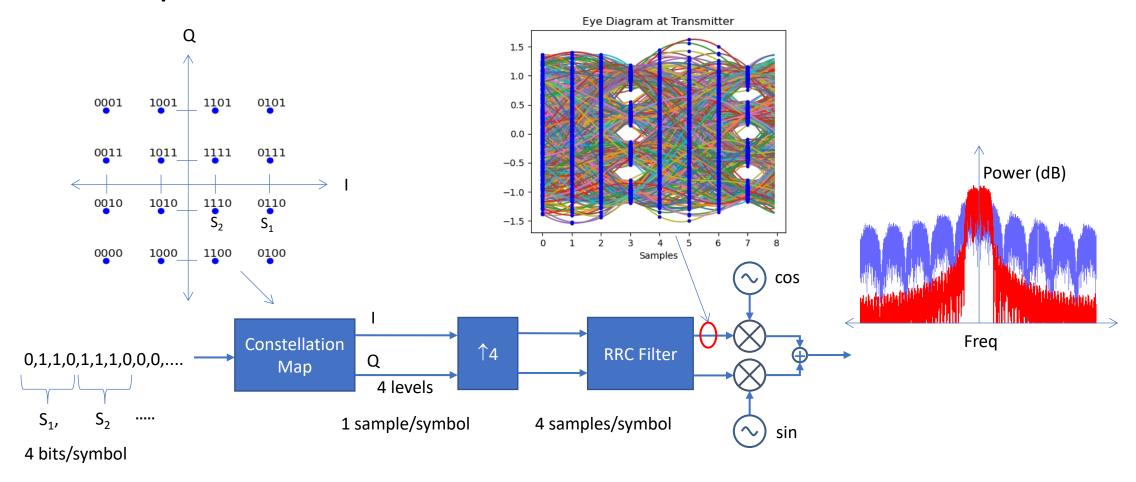
$$\rho = \frac{\operatorname{Cov}(V, \widehat{V})}{\sigma_V \sigma_{\widehat{V}}}$$

$$\rho = \frac{\frac{1}{N} \sum \left((V - \bar{V}) \left(\hat{V} - \bar{\hat{V}} \right)^* \right)}{\sqrt{\frac{1}{N} \sum |V - \bar{V}|^2} \sqrt{\frac{1}{N} \sum \left| \hat{V} - \bar{\hat{V}} \right|^2}} = \frac{\sum \left((V - \bar{V}) \left(\hat{V} - \bar{\hat{V}} \right)^* \right)}{\sqrt{\sum \sum |V - \bar{V}|^2} \sqrt{\sum \left| \hat{V} - \bar{\hat{V}} \right|^2}}$$

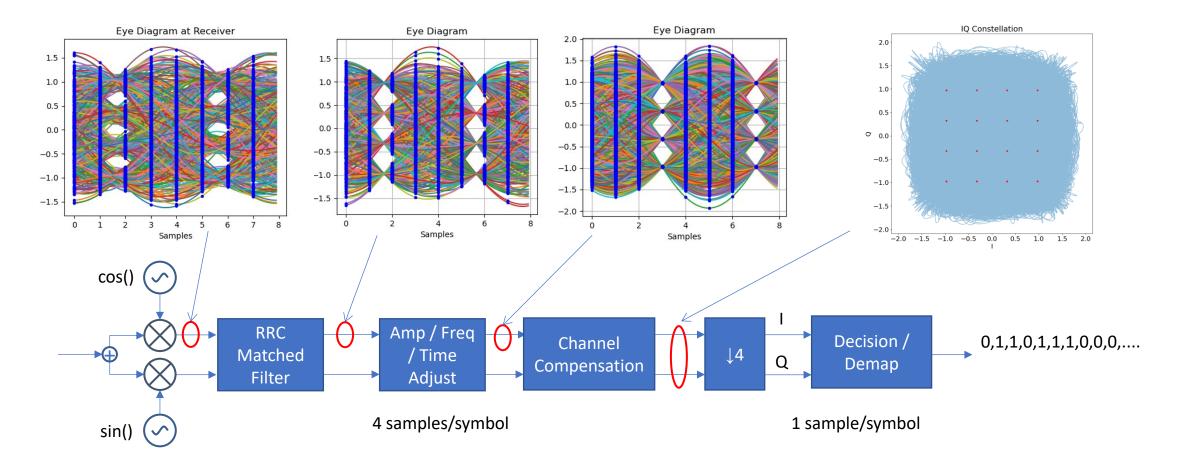
If zero mean:
$$\rho = \frac{\sum (V\hat{V}^*)}{\sqrt{\sum |V|^2} \sqrt{\sum |\hat{V}|^2}}$$

$$SNR_{dB} = 10 \log_{10} \frac{|\rho|^2}{1 - |\rho|^2}$$

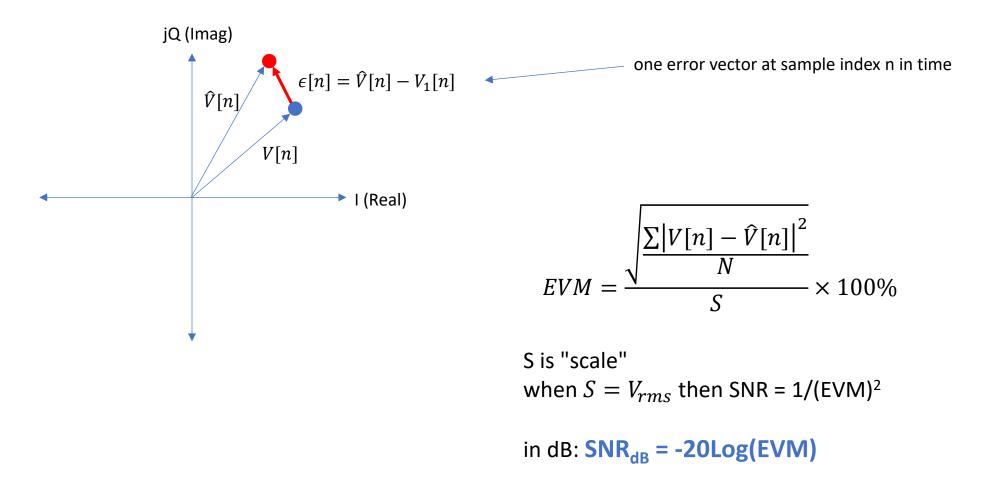
Example Waveform: 16-QAM Transmitter



Example Waveform: 16-QAM Receiver



EVM: Error Vector Magnitude



16-QAM Waveform Parameters

Total number of symbols: 262,144

Symbol Rate (R): 10 MHz

Headroom: 8 dB below Full Scale

Number of bits: 8 bits I and Q

Sampling Rate (f_s): 40 MHz

RRC impulse response duration: 30 symbols

RRC roll-off factor: 0.2

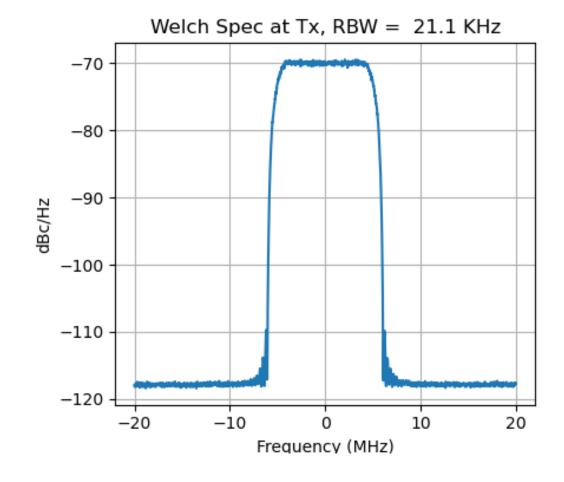
Predicted SNR as limited by quantization noise:

SNR (from Full Scale) = 6.02 dB/bit + 1.76 dB = (6.02)(8) + 1.76 = 49.92 dB

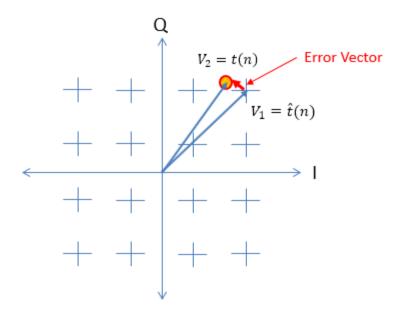
SNR after Headroom = 49.92 dB - 8 dB = 41.92 dB

Potential SNR increase due to oversampling = $10\log(f_s/R) = 6.02 \text{ dB}$

SNR = 41.92 dB + 6.02 dB = 47.94 dB



SNR using EVM



$$EVM = \frac{\sqrt{\frac{\sum |t_n - \hat{t}_n|^2}{N}}}{S} \times 100\%$$

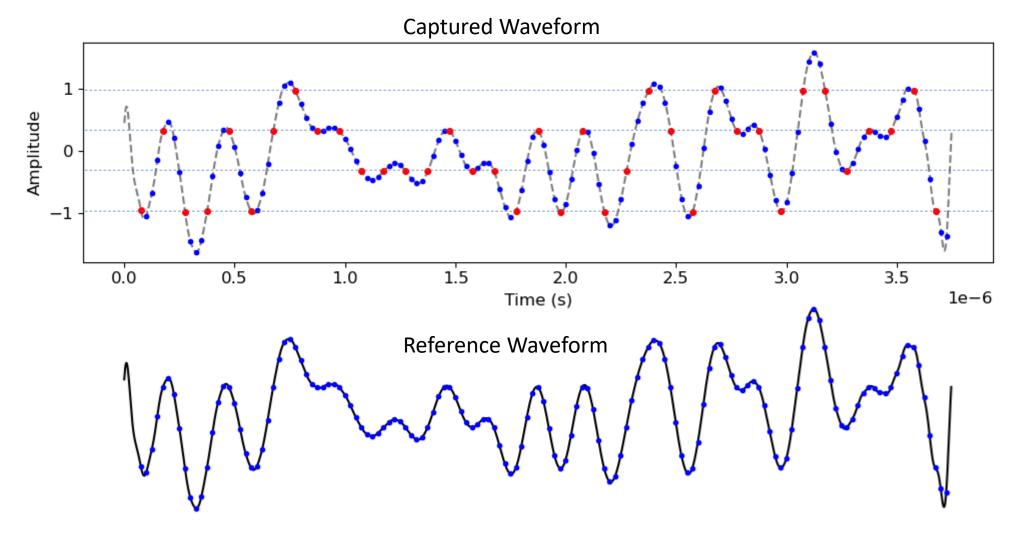
When S = RMS of all possible symbols in the constellation

then:

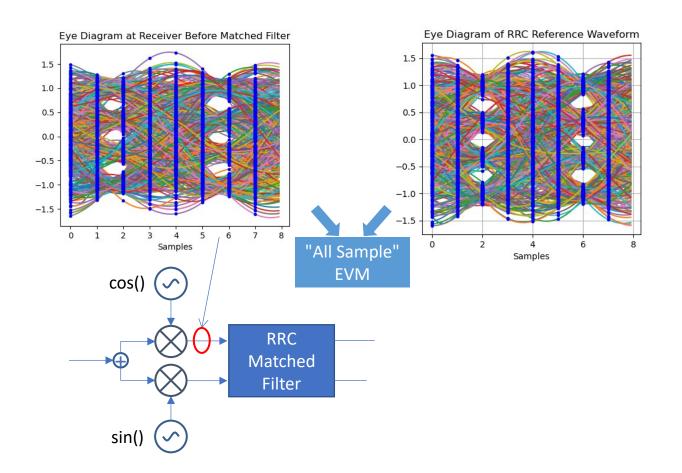
 $1/EVM^2 = SNR$

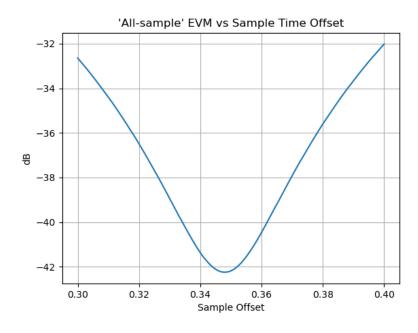
in dB: SNR (dB) = -20Log(EVM)

"All Sample" EVM



EVM for 16-QAM prior to Matched Filter





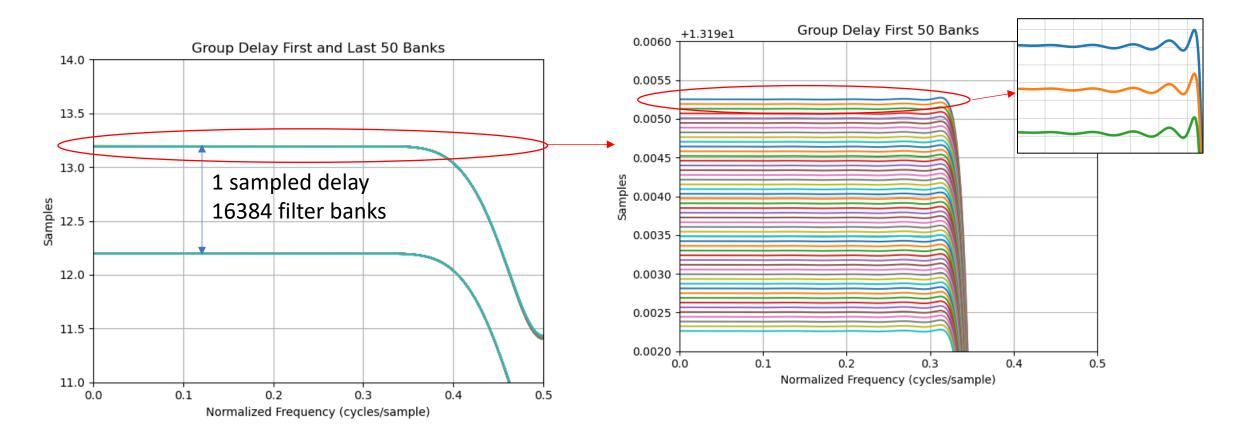
Alignment of Mag, Freq/Phase, and Time

Magnitude: Normalize by the standard deviation

Freq / Phase: Use correlation to get phase, frequency is slope of phase

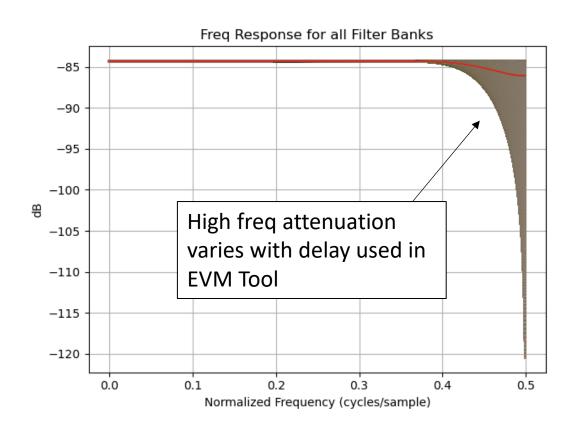
Time: Use correlation for course alignment, polyphase for fine adjust

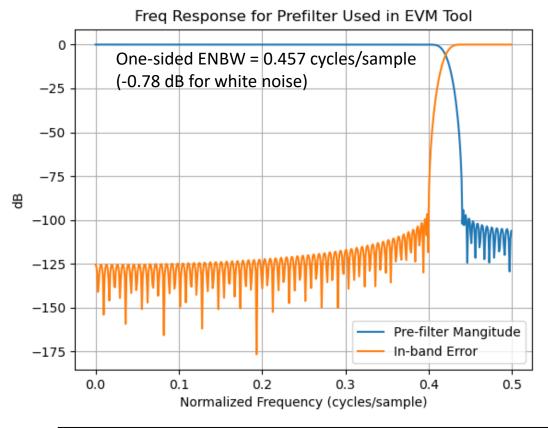
Time Alignment with Polyphase Filter Bank



Each Bank is a 27 tap FIR filter

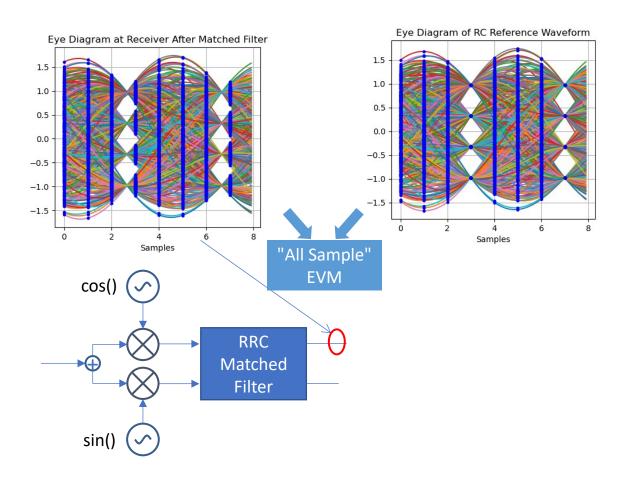
Magnitude Response of Polyphase Filter Bank

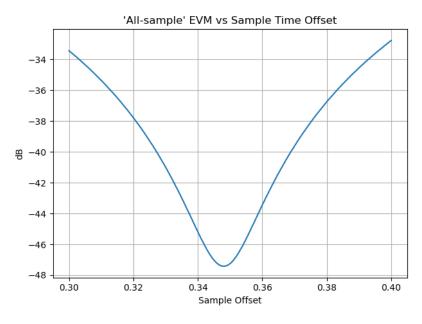




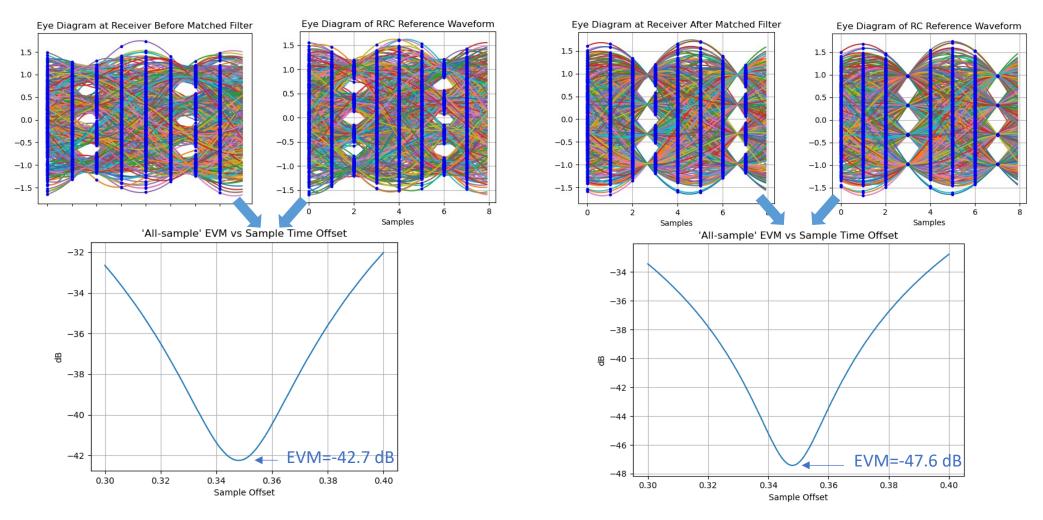
Prefilter for waveform and reference used in EVM Tool to eliminate delay variability in EVM result

EVM for 16-QAM after Matched Filter





Summary of EVM Receiver Measurements





Rx Waveform Measurement Before Matched Filter:

In: tools.evm(rx_before_mf, reference_rrc)

Out: (0.007323, 0.347961, 4.519621e-07)

In: tools.db(0.007323)

Out: -42.7062

EVM, time offset (samples), and phase offset (radians)

◆ -42.7 dB

Rx Waveform Measurement After Matched Filter:

In: tools.evm(rx_after_mf, reference_rc)
Out: (0.004149, 0.348022, 8.341614e-08)

In: tools.db(0.004149)

Out: -47.64107

-47.6 dB

Measured EVM at Decision Samples Only: -47.9 dB

Predicted SNRs – 16 QAM

16 QAM SNR Prior to Matched filter

Quantization: 8 bits I and Q

AGC: -8.0 dBFS

Raised Cosine Roll-off Factor (α): 0.2

One-sided ENBW of Prefilter in EVM Tool: 0.457 cycles/sample

Sampling Rate (R): 4 samples / symbol

SNR using 6.02 dB/bit + 1.76 dB: 49.92 dBFS

SNR after AGC Back-off: 49.92 dBFS - 8.0 dB = 41.92 dBPrefilter noise reduction: $20\text{Log}(2 \times \text{ENBW}) = -0.78 \text{ dB}$

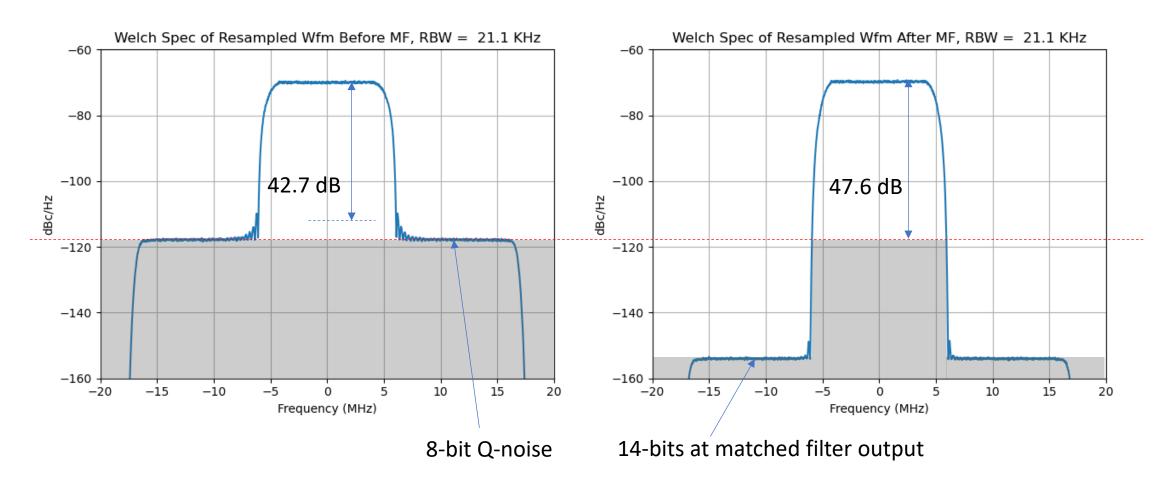
Predicted "All-Sample" EVM from EVM Tool: -41.92 dB - 0.78 dB = -42.70 dB

16 QAM SNR After Matched filter

Potential SNR increase due to oversampling: $20 \log_{10}(R) = 6.02 \text{ dB}$

Predicted EVM after 2nd RRC fitler (removes out of band noise): -41.92-6.02 =-47.94 dB

Measured SNRs and Welch Spectrums



SNR using the Correlation Coefficient

Step 1: Remove time, frequency, phase offsets.

Step 2: Remove the mean.

Step 3: Normalize by the standard deviation to get waveform x[n] and reference r[n].

Compute the normalized correlation coefficient using same length N for x[n] and r[n]:

$$\rho = \frac{1}{N} \sum_{N} x[n] r^*[n]$$

$$SNR_{dB} = 10 log_{10} \frac{|\rho|^2}{1 - |\rho|^2}$$

SNR from EVM and Correlation – 16 QAM

Using EVM:

$$EVM = \sqrt{\frac{\sum_{N} |x[n] - r[n]|^2}{N}}$$

$$SNR_{dB} = -20\log_{10}(EVM)$$

SNR after Matched Filter for 16-QAM Example:

All samples: -47.6 dB

Decision samples only: -47.9 dB

Using Correlation Coefficient:

$$\rho = \frac{\sum_{N} x[n]r^*[n]}{N}$$

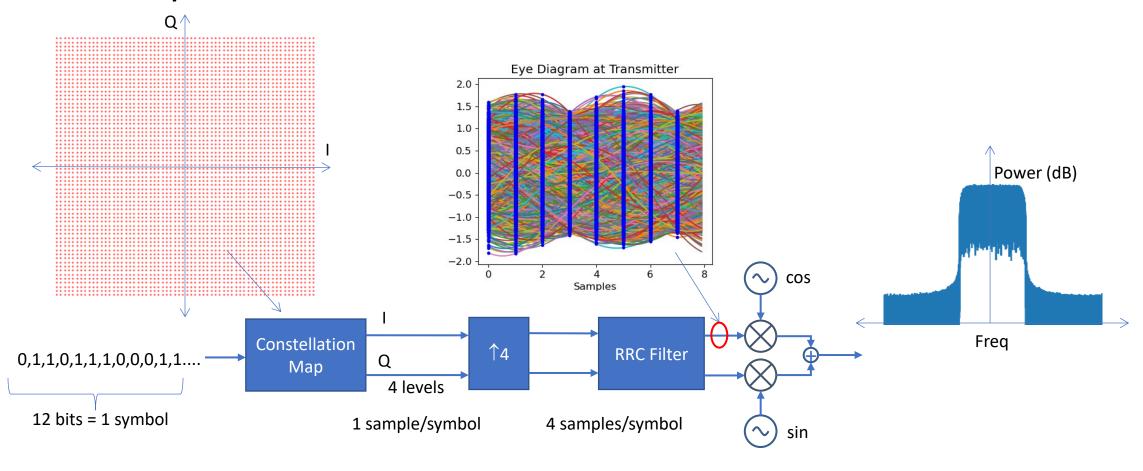
$$SNR_{dB} = 10 \log_{10} \frac{|\rho|^2}{1 - |\rho|^2}$$

SNR after Matched Filter for 16-QAM Example:

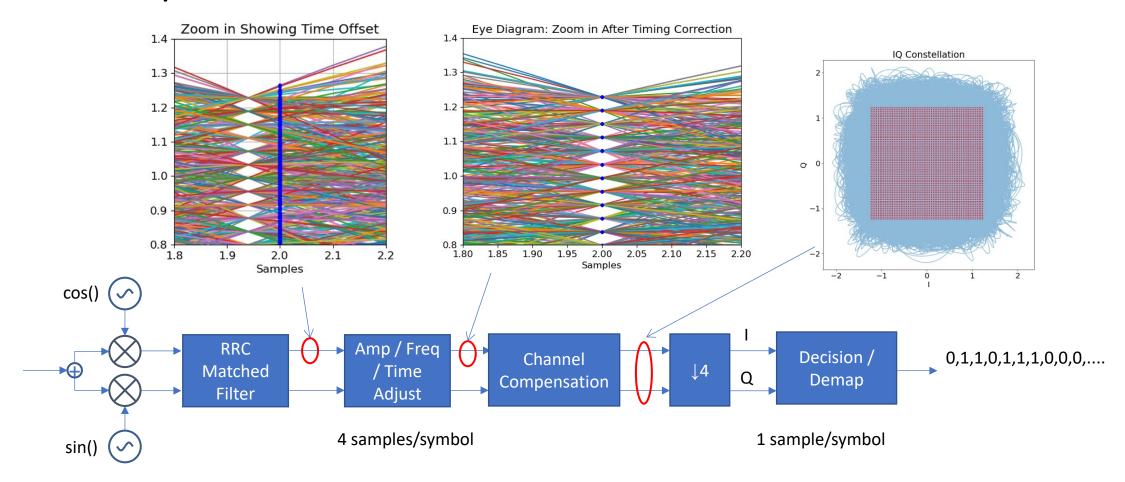
All samples: -47.6 dB

Decision samples only: -47.9 dB

Example Waveform: 4096-QAM Transmitter



Example Waveform: 4096-QAM Receiver



4096-QAM Waveform Parameters

Total number of symbols: 262,144

Symbol Rate (R): 1 MHz

Headroom: 8 dB below Full Scale

Number of bits: 15 bits I and Q (SOTA = 15 bits)

Sampling Rate (f_s): 8 MHz

RRC impulse response duration: 400 symbols

RRC roll-off factor: 0.2

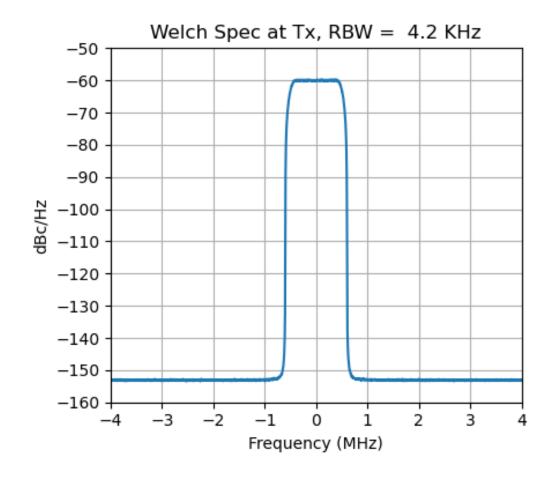
Predicted SNR as limited by quantization noise:

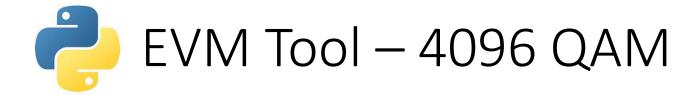
SNR (from Full Scale) = $6.02 \, dB/bit + 1.76 \, dB = (6.02)(15) + 1.76 = 92.06 \, dB$

SNR after Headroom = 104.10 dB - 8 dB = 84.06 dB

Potential SNR increase due to oversampling = $10\log(f_s/R) = 9.03 \text{ dB}$







Rx Waveform Measurement Before Matched Filter:

In: tools.evm(rx_before_mf, reference_rrc)

Out: (5.689957e-05, 0.059998, 9.174326e-10)

In: tools.db(5.689957e-05)

Out: -84.8978

◆ EVM, time offset (samples), and phase offset (radians)

◆ -84.9 dB

Rx Waveform Measurement After Matched Filter:

In: tools.evm(rx_after_mf, reference_rc)

Out: (2.276750e-05, 0.059998, -4.41151e-08)

In: tools.db(2.276750e-05)

Out: -92.8537

-92.9 dB

Measured EVM at Decision Samples Only: -93.0 dB

Predicted SNRs – 4096 QAM

4096 QAM SNR Prior to Matched filter

Quantization: 15 bits I and Q

AGC: -8.0 dBFS

Raised Cosine Roll-off Factor (α): 0.2

One-sided ENBW of Prefilter in EVM Tool: 0.457 cycles/sample

Sampling Rate (R): 8 samples / symbol

SNR using 6.02 dB/bit + 1.76 dB: 92.06 dBFS

SNR after AGC Back-off: 92.06 dBFS - 8.0 dB = 84.06 dBPrefilter noise reduction: $20\text{Log}(2 \times \text{ENBW}) = -0.78 \text{ dB}$

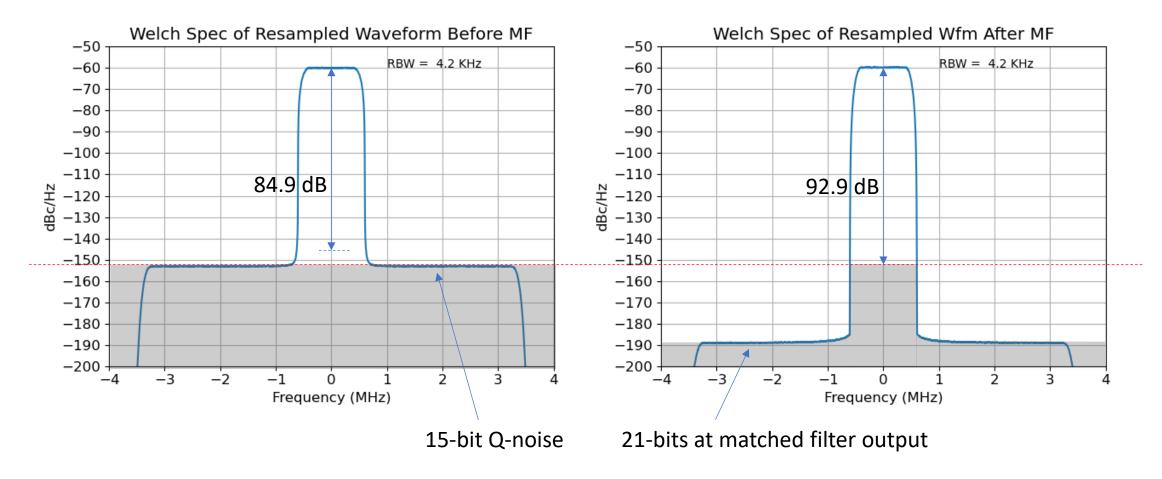
Predicted "All-Sample" EVM from EVM Tool: -84.06 dB - 0.78 dB = 84.84 dB

4096 QAM SNR After Matched filter

Potential SNR increase due to oversampling: $20 \log_{10}(R) = 9.03 \text{ dB}$

Predicted EVM after 2nd RRC fitler (removes out of band noise): 84.06+9.03 =93.09 dB

Measured SNRs and Welch Spectrums



SNR from EVM and Correlation – 4096 QAM

Using EVM:

$$EVM = \sqrt{\frac{\sum_{N} |x[n] - r[n]|^2}{N}}$$

$$SNR_{dB} = -20\log_{10}(EVM)$$

SNR after Matched Filter for 4096-QAM Example:

All samples: -92.9 dB

Decision samples only: -93.0 dB

Using Correlation Coefficient:

$$\rho = \frac{\sum_{N} x[n]r^*[n]}{N}$$

$$SNR_{dB} = 10 \log_{10} \frac{|\rho|^2}{1 - |\rho|^2}$$

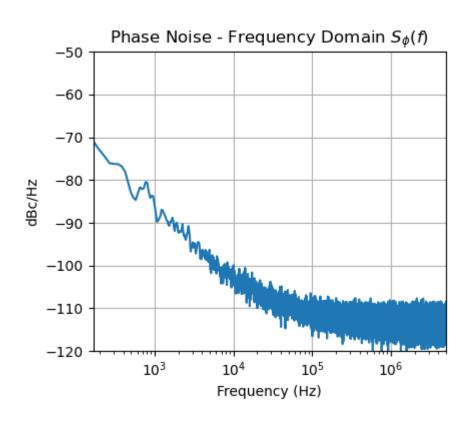
SNR after Matched Filter for 4096-QAM Example:

All samples: -92.9 dB

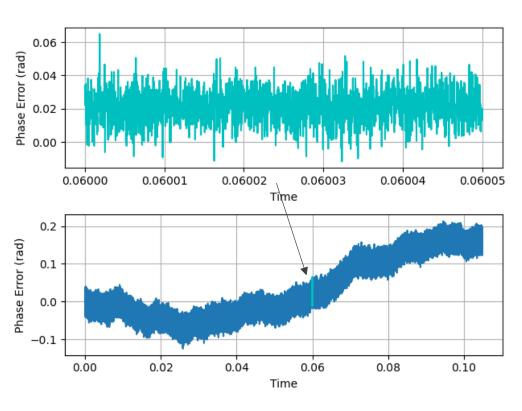
Decision samples only: -93.0 dB

Non-Stationary Signals

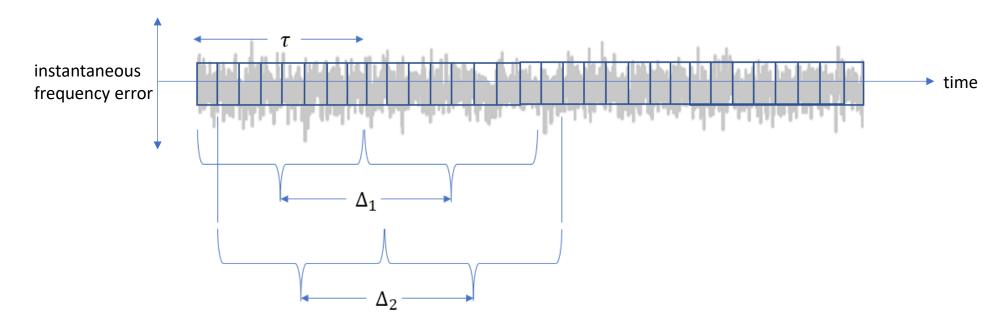
Consideration for Non-Stationary Signals



Phase Noise – Time Domain



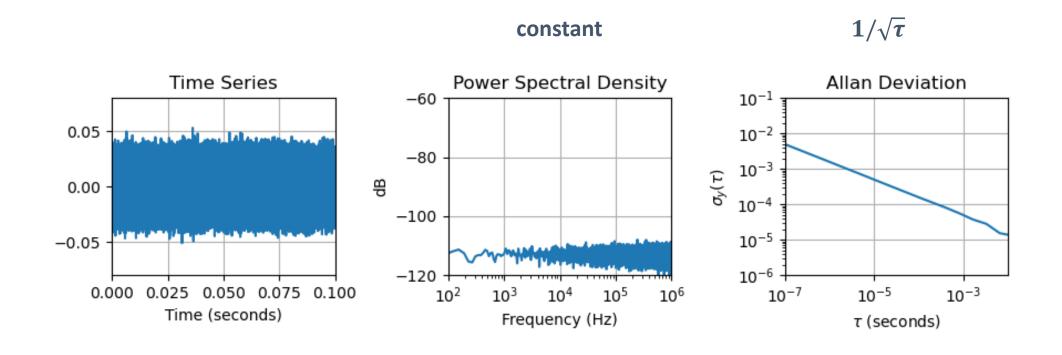
Allan Deviation (ADEV)



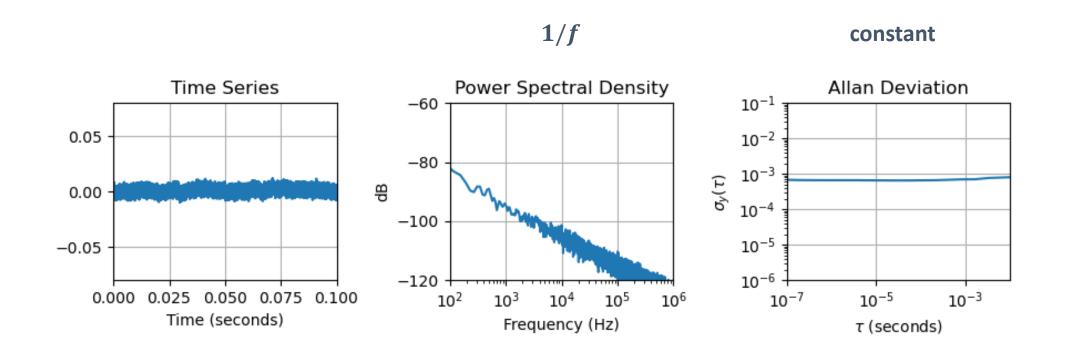
Overlap ADEV:

Average the frequency error over duration $\ au$ Shift one sample, compute a new average for every shift Subtract averages that are $\ au$ apart in time to get a difference error RMS all the differences Normalize answer by dividing by $\sqrt{2}$

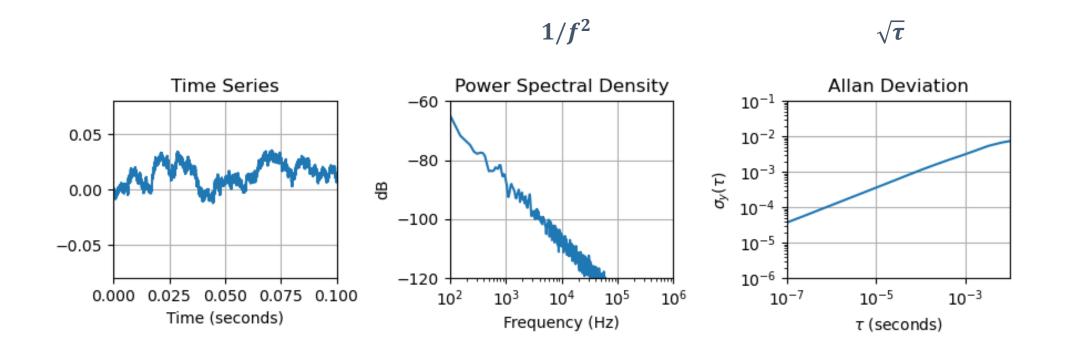
ADEV – White Noise



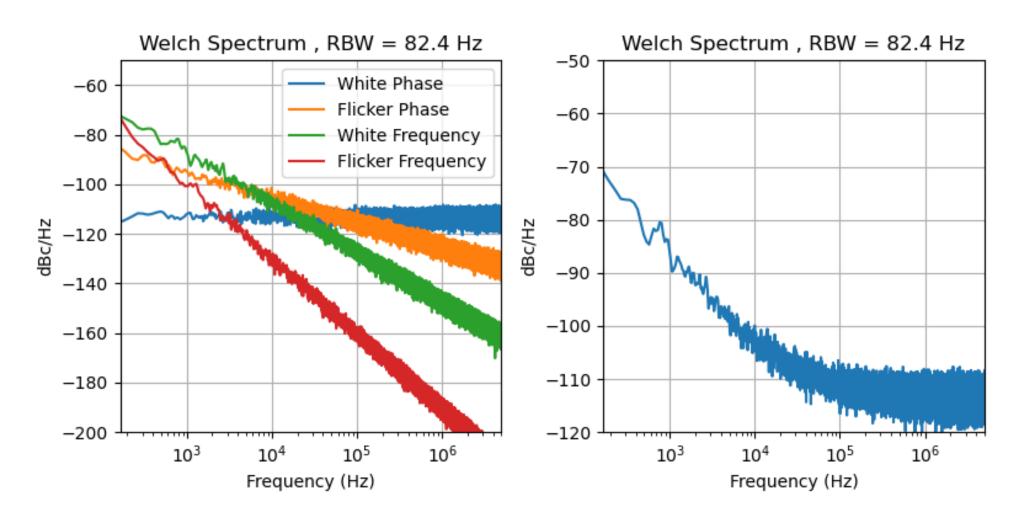
ADEV – Pink Noise



ADEV – Red (Brownian) Noise

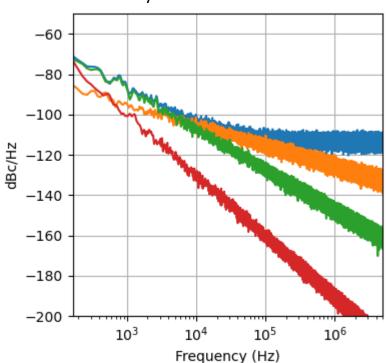


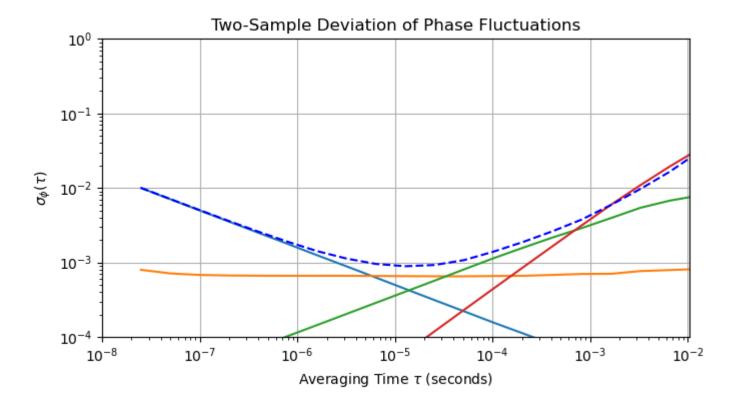
Phase Power Spectral Density - Components



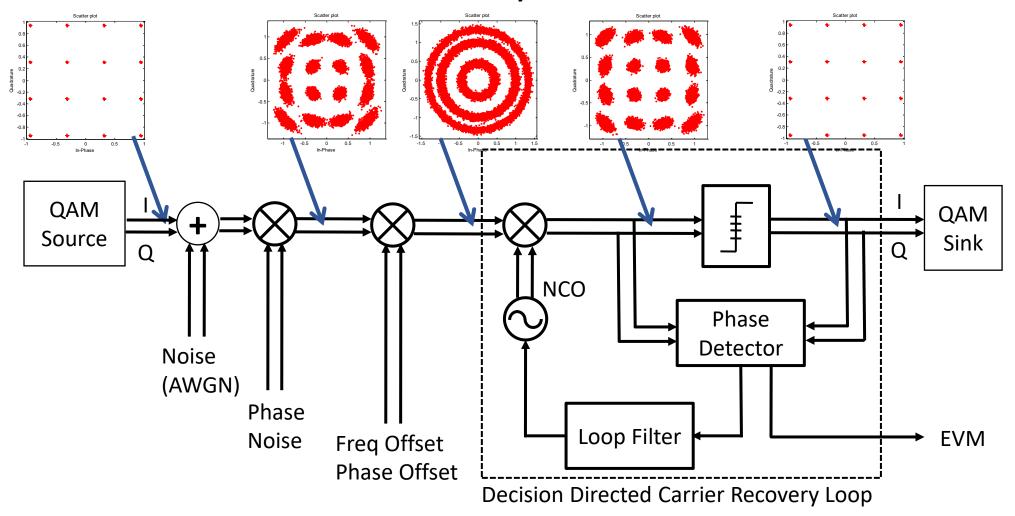
"ADEV" for Phase Fluctuations

 $S_{\phi}(f)$ Two-Sided Power Spectral Density due to Phase Fluctuations



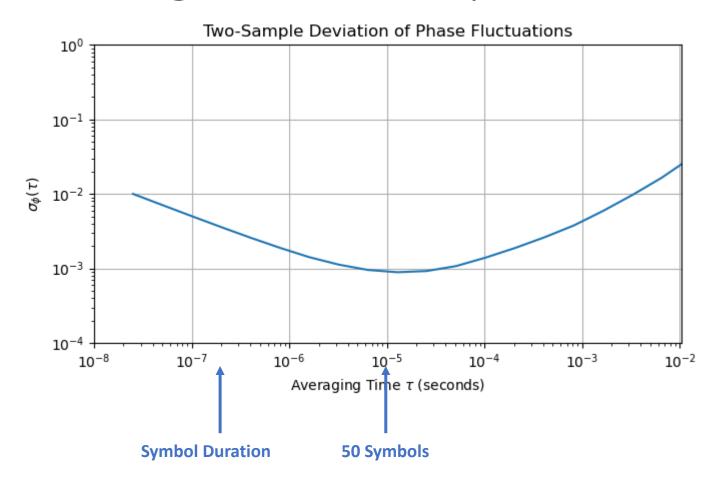


QAM Carrier Recovery and Phase Noise

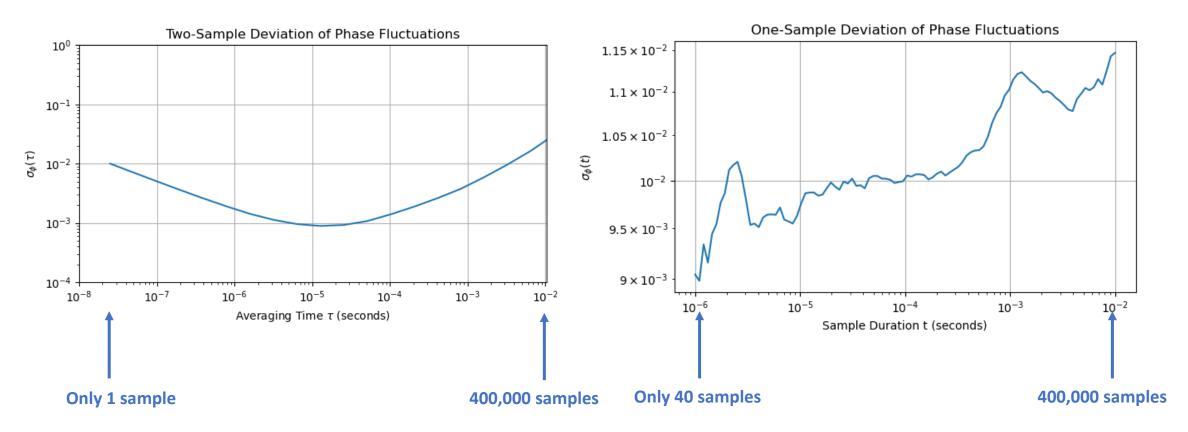


QAM Example Showing ADEV Utility

4096 QAM Symbol Rate 5 MHz (duration 200 ns) Sample Rate 40 MHz RC alpha = 0.25



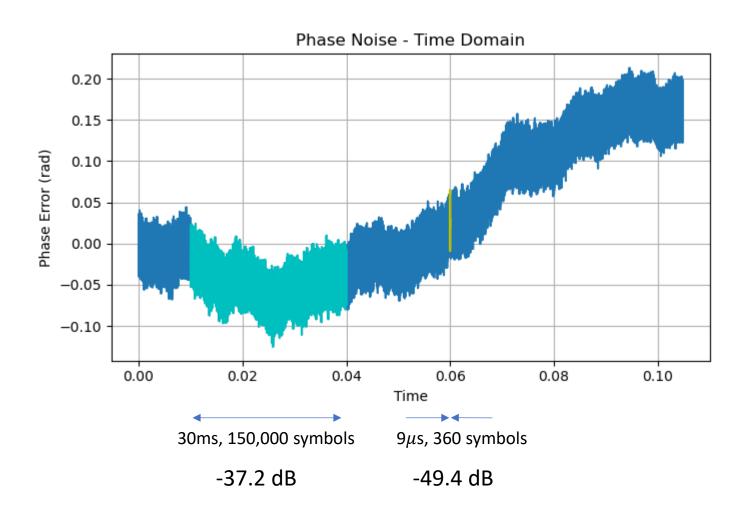
"Two-Sample" vs "One-Sample" Deviation



- 1) Avg each block
- 2) std of the differences between block avgs

- 1) std of each block
- 2) avg the stds of each block

4096 QAM EVM Results



Recap For Future Reference

FFT Spectrum to measure Tones

Zero pad FFT 4x to 10x to minimize "scalloping loss" (next pow2)
Window with Kaiser Window, Adjust beta to adjust RBW
Scale FFT by sum of the window for accurate tone measurement

```
N = len(x)
win = sig.windows.kaiser(N, beta)
fout = fft.fft(x * win, N * 10) / np.sum(win)
```

Understand Resolution Bandwidth

```
rbw = len(win) * np.sum(win**2) / np.sum(win)**2 # rbw in bins
```

Welch Spectrum to measure Spectral Density

Primary knobs

Block size (nperseg): trades resolution BW and resolution

Window (win): eliminates spectral leakage

Block overlap (noverlap): minimize double counting: 50% for Hann, 75% for Kaiser

Additional details

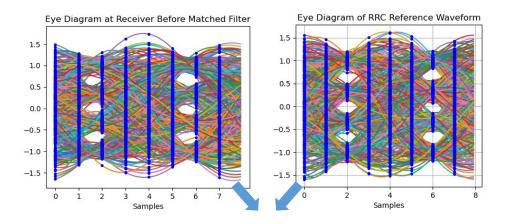
One-sided or two-sided (return_onesided)

Average with 'mean' vs 'median'

Detrend (I prefer False)

Returns a **Power Quantity** (use 10log10() to convert to dB)

EVM, Correlation and SNR



x: waveformr: reference(both mean removed and normalized by their std)

Using **EVM**:

$$EVM = \sqrt{\frac{\sum_{N} |x[n] - r[n]|^2}{N}}$$

$$SNR_{dB} = -20\log_{10}(EVM)$$

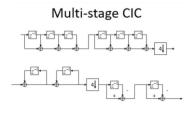
Using Correlation Coefficient:

$$\rho = \frac{\sum_{N} x[n]r^*[n]}{N}$$

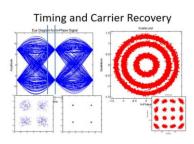
$$SNR_{dB} = 10 \log_{10} \frac{|\rho|^2}{1 - |\rho|^2}$$

Want More Signal Processing??

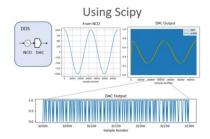
https://dsp-coach.com



Signal Processing for Wireless Communications

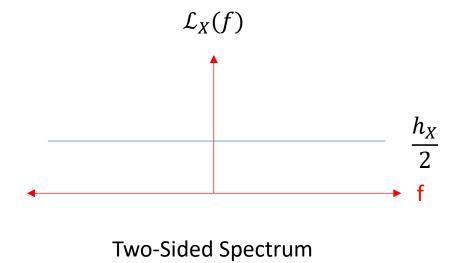


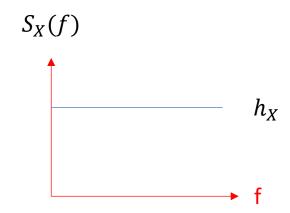
Signal Processing for Software Radio



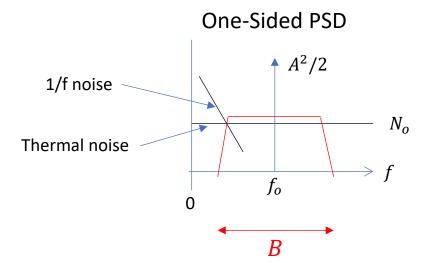
Python Applications for Digital Design and Signal Processing

Backup Slides

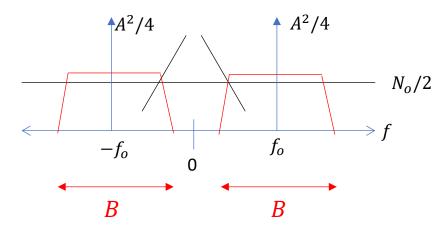




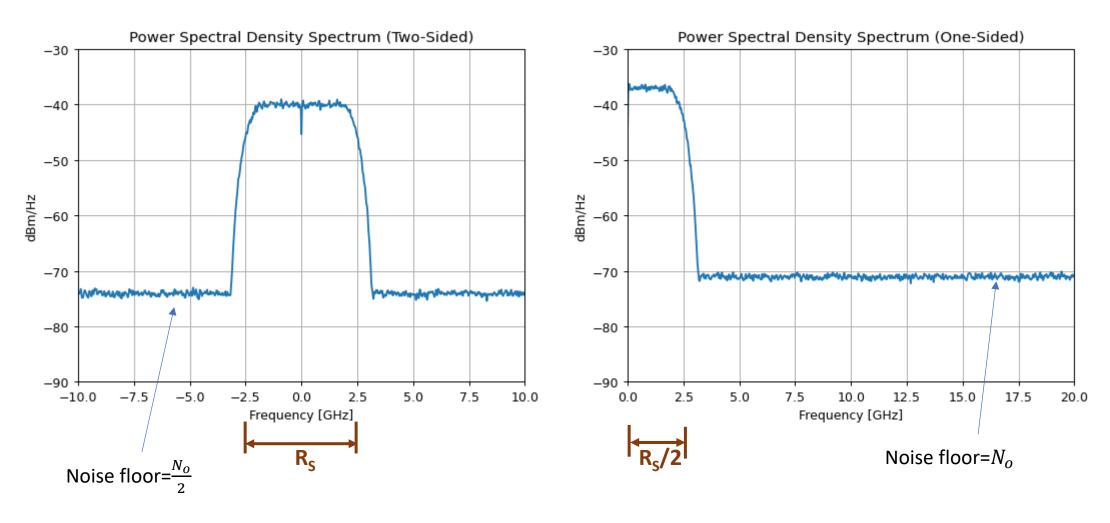
One-Sided Spectrum

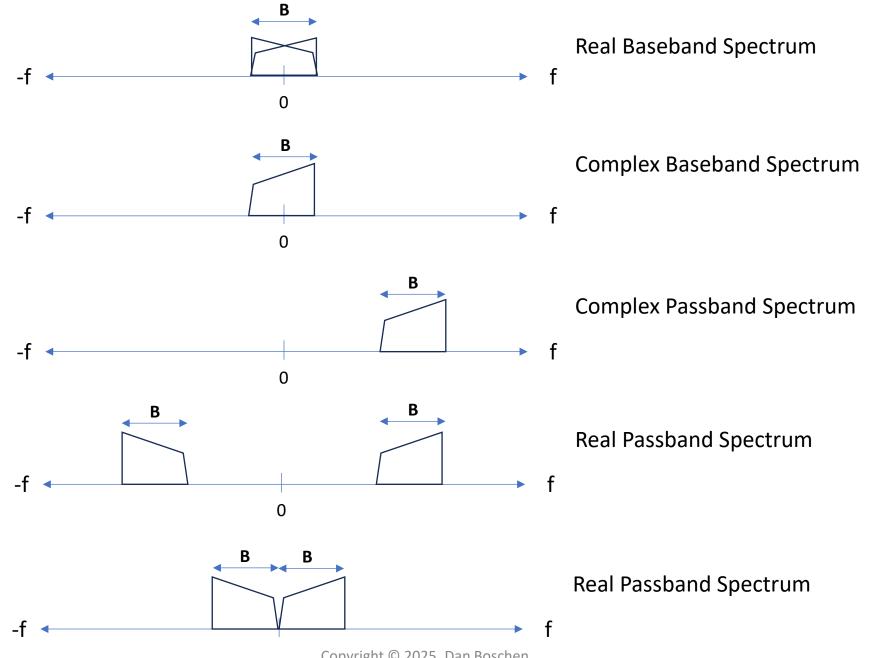


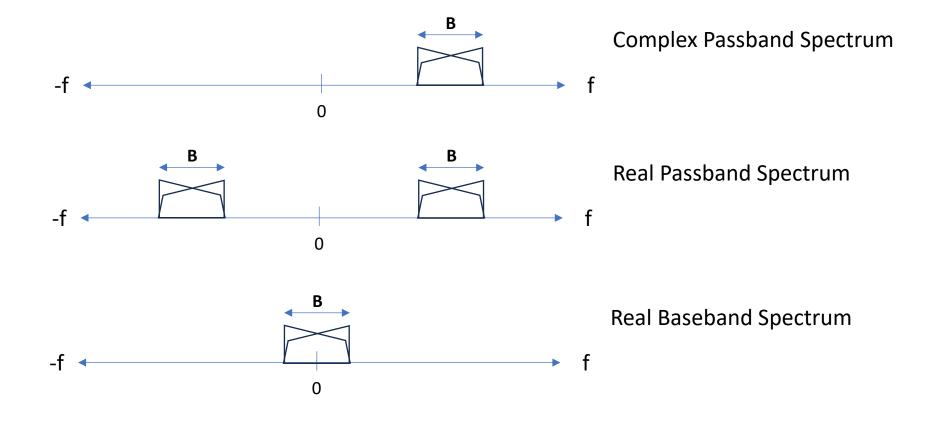
Two-Sided PSD

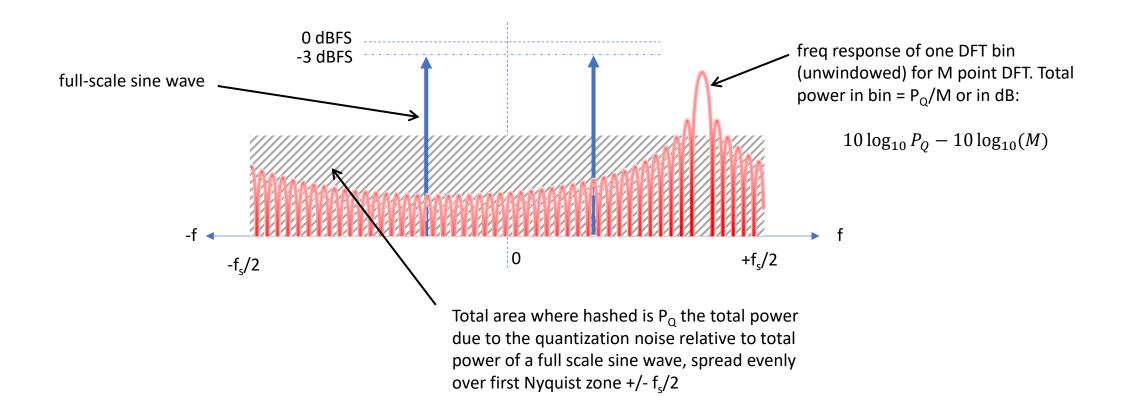


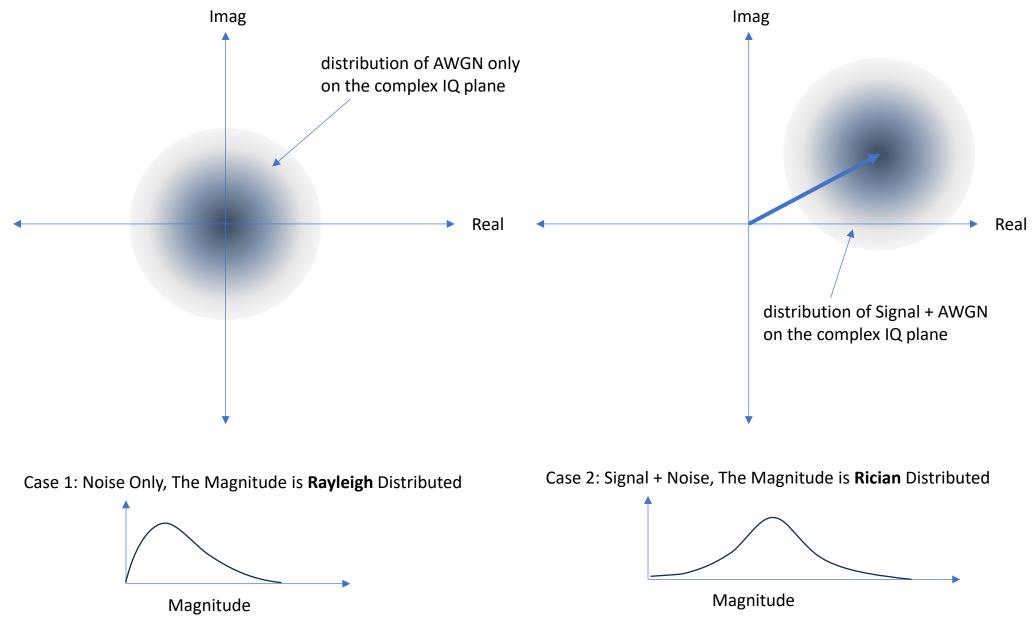
QPSK symbol rate: R_s =5 GSymbols/sec (Raised Cosine $\alpha=0.3$)

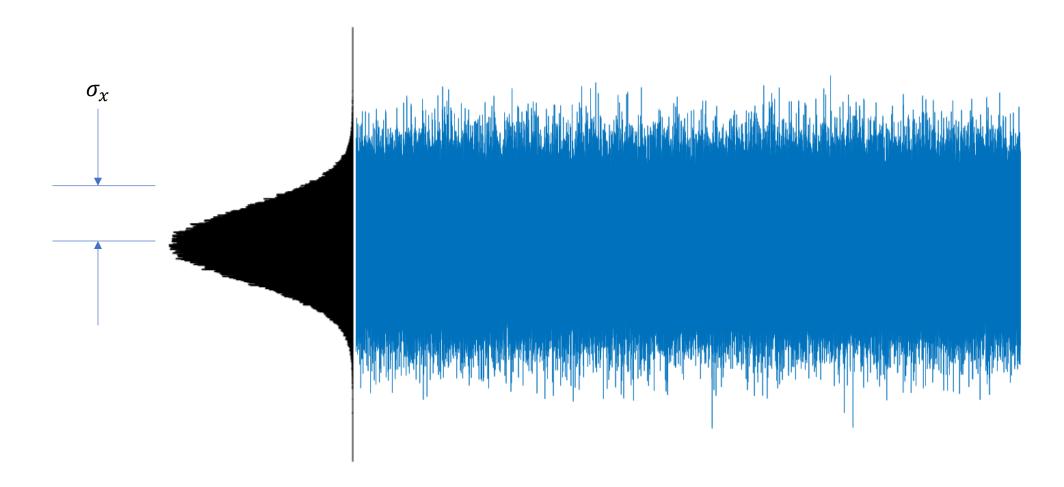




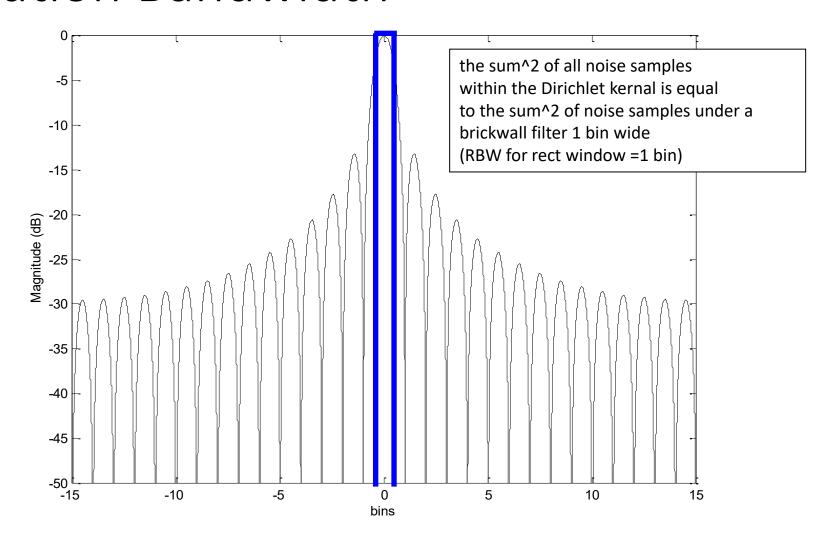




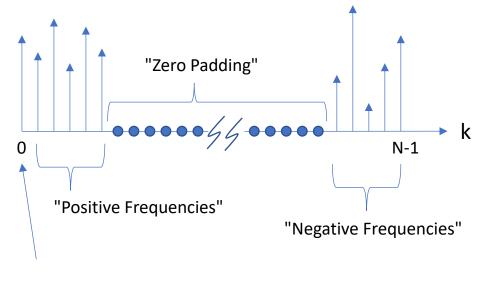




Resolution Bandwidth

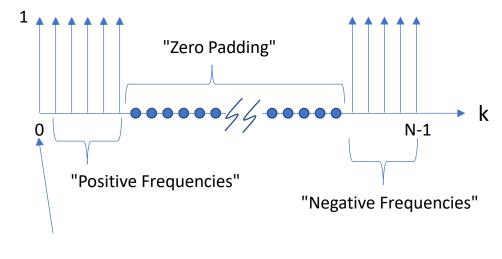


DFT Magnitude

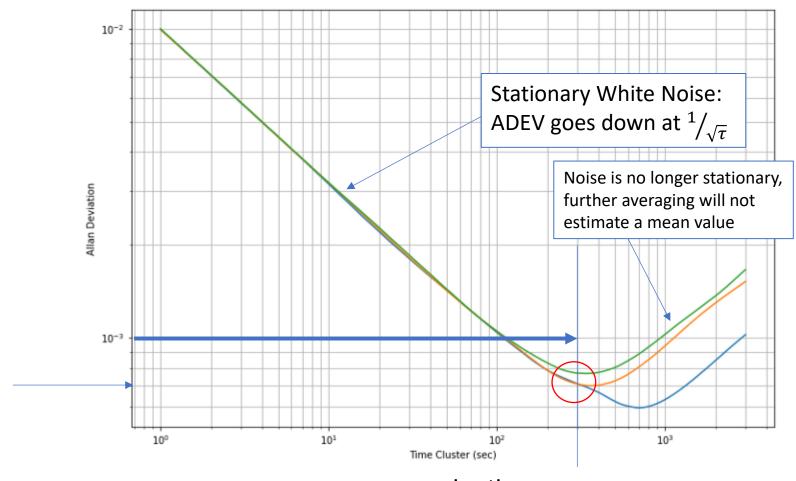


"DC Bin"

DFT Window

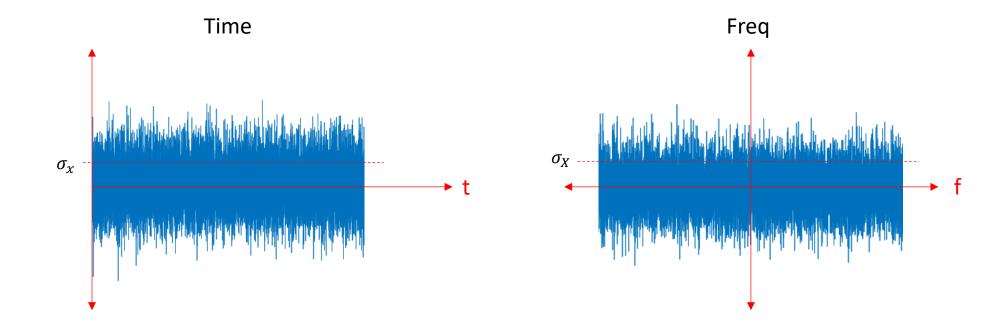


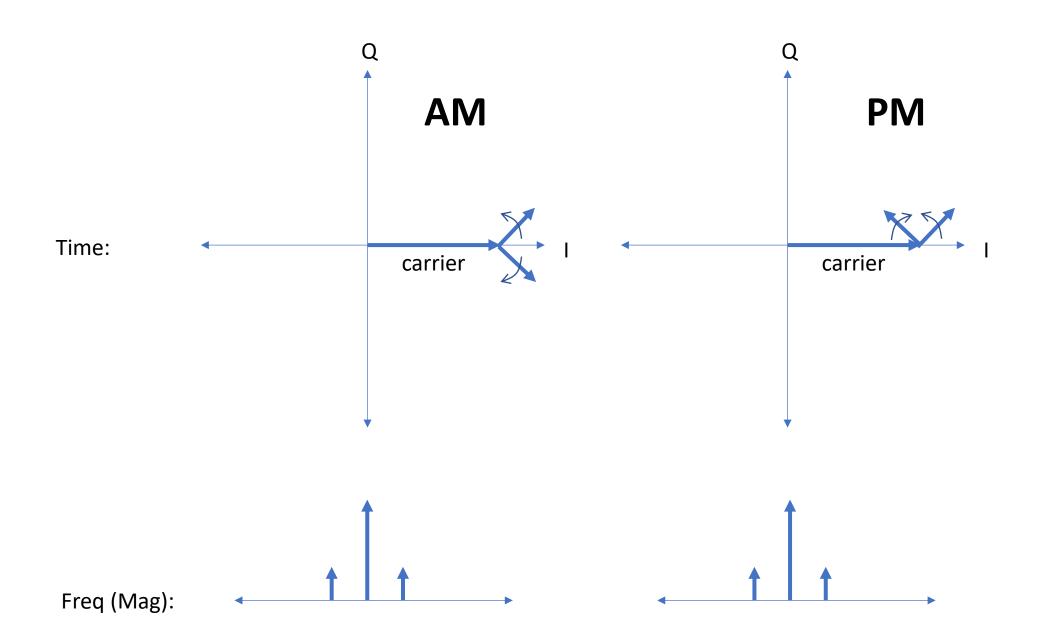
"DC Bin"



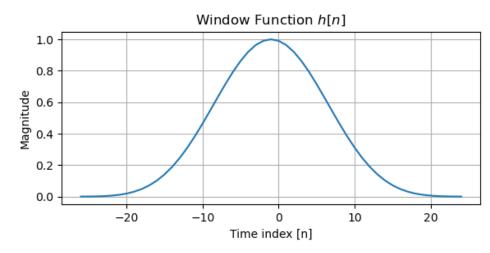
For a "white noise" process this will be the standard deviation of the estimated mean when averaged for τ seconds (in this case of the orange curve, 300 seconds)

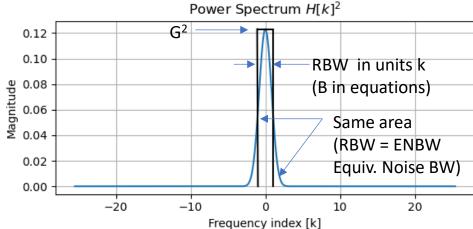
 τ : averaging time





Resolution BW (RBW)





Given scaled DFT as
$$H[k] = \frac{1}{N} \sum_{n=0}^{N-1} h[n] W_N^{nk}$$

$$\frac{1}{N}\sum h[n]^2 = \sum |H[k]|^2$$

(1) Parseval's Theorem

$$G = \frac{1}{N} \sum h[n]$$

(2) DC Gain of window (DC bin in DFT)

$$\sum |H[k]|^2 = BG^2$$

(3) Area under Power Spectrum of window = Area under equivalent brickwall filter

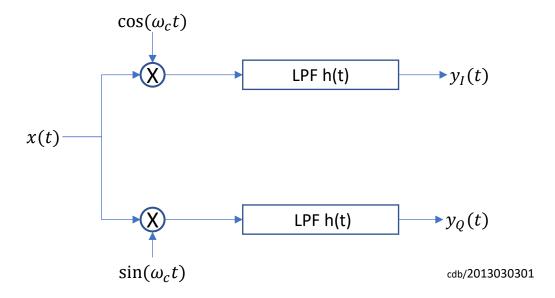
Combining (1), (2), and (3):

$$B\left(\frac{1}{N}\sum h[n]\right)^2 = \frac{1}{N}\sum h[n]^2$$

Solve for B:

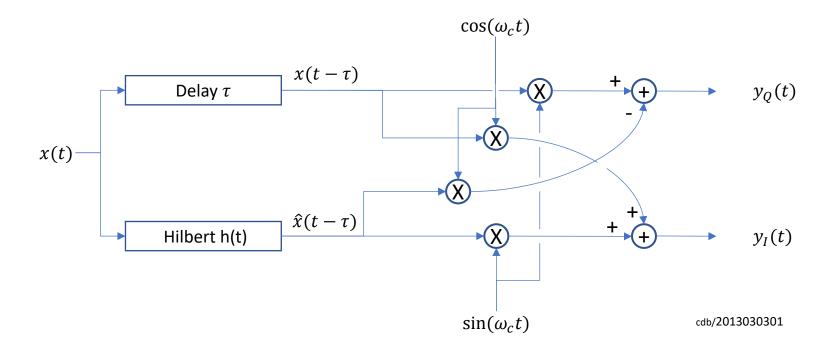
$$B = N \frac{\sum h[n]^2}{(\sum h[n])^2}$$

Method 1



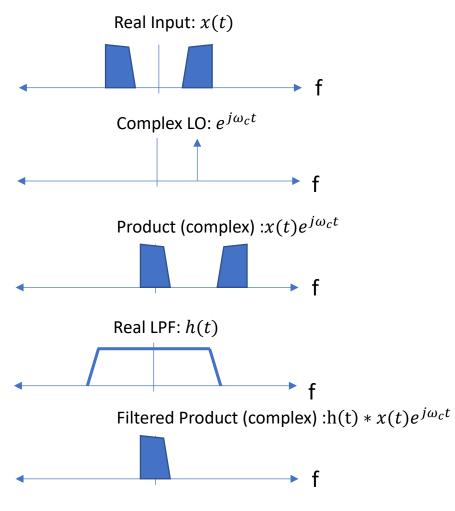
Methods for translating Real IF to Complex Baseband I and Q

Method 2



Methods for translating Real IF to Complex Baseband I and Q

Spectrums for "Method 1"



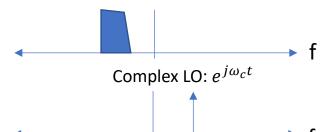
cdb/2013030301

Spectrums for "Method 2"

Real Input: x(t)



Complex Input: $x(t) - j\hat{x}(t)$

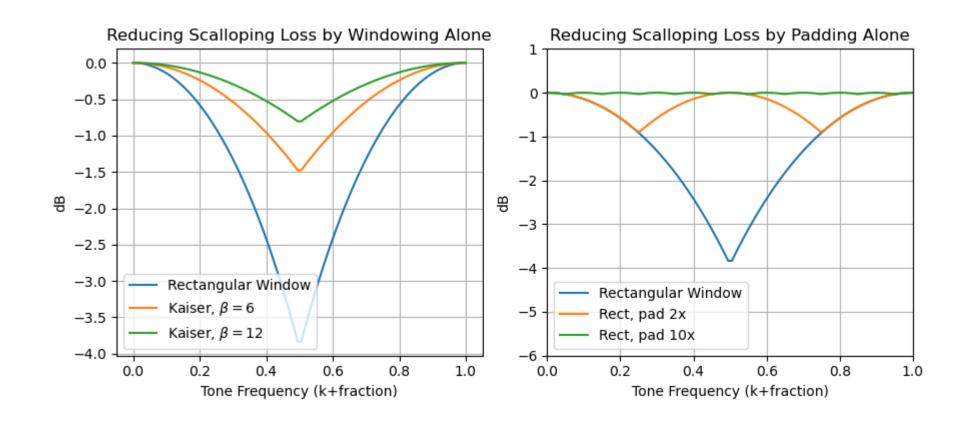


Product (complex) : $(x(t) - j\hat{x}(t))e^{j\omega_c t}$



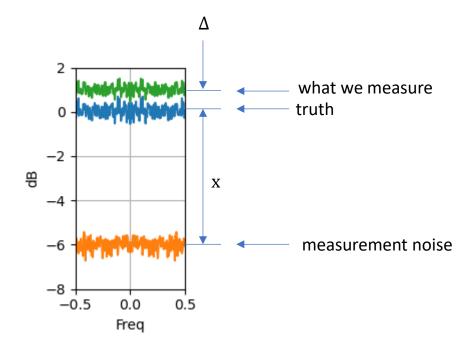
cdb/2013030301

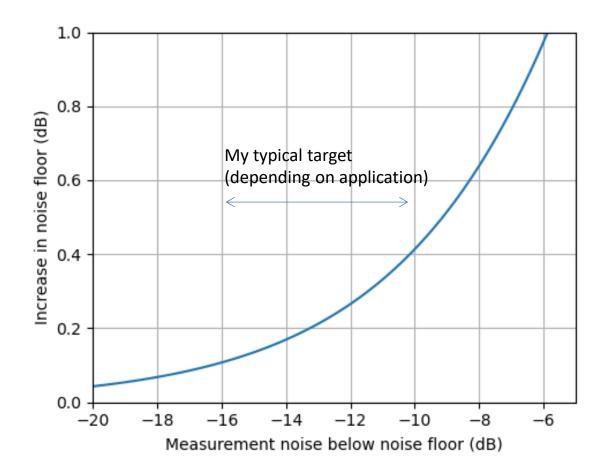
Scalloping Loss



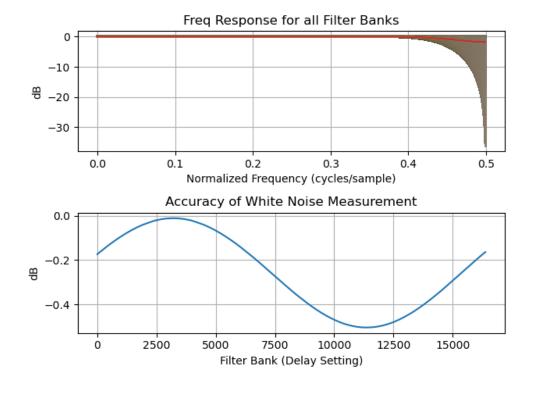
Measurement Noise

$$\Delta = 10 \log_{10} (1 + 10^{x/10})$$

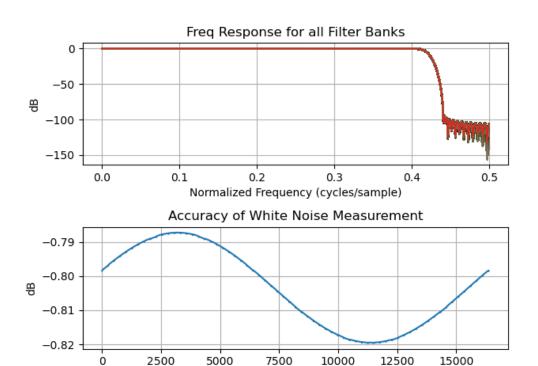




Error in Noise Measurement for White Noise vs Delay Adjustment with and without Prefilter in EVM Tool



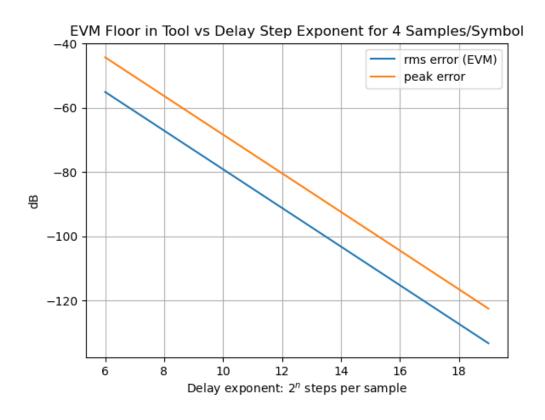
Without Prefilter
0.5 dB variation, minimum offset

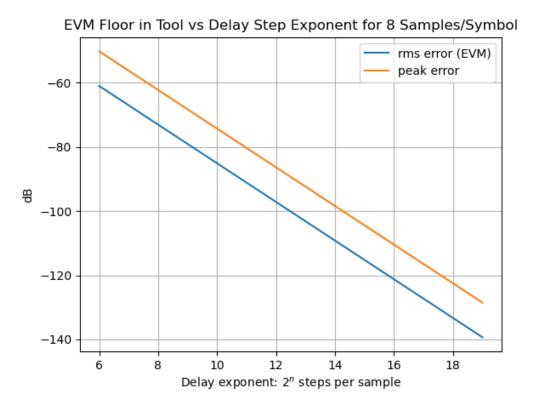


With Prefilter
0.035 dB variation, fixed offset for white noise
(Inconsequential for waveforms that occupy <80 % of Nyquist)

Filter Bank (Delay Setting)

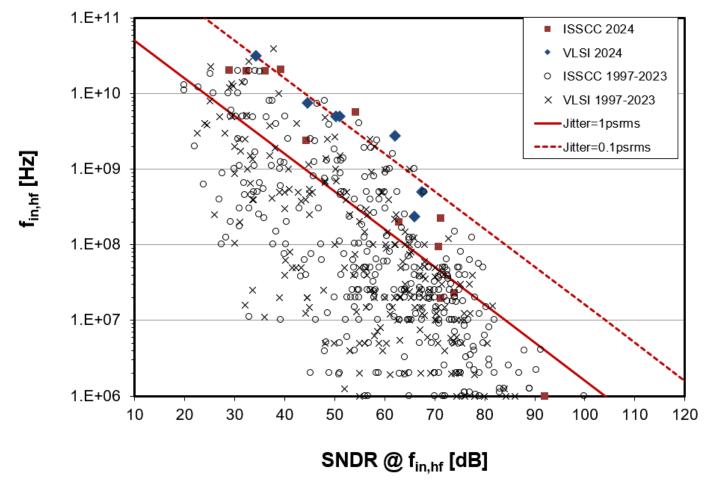
What time step precision is needed?



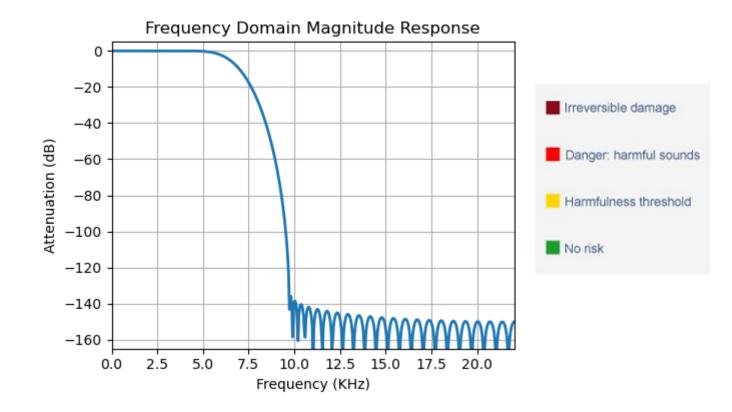


ADC SOTA

"B. Murmann, "ADC Performance Survey 1997-2024," [Online]. Available: https://github.com/bmurmann/ADC-survey."



dB Scale



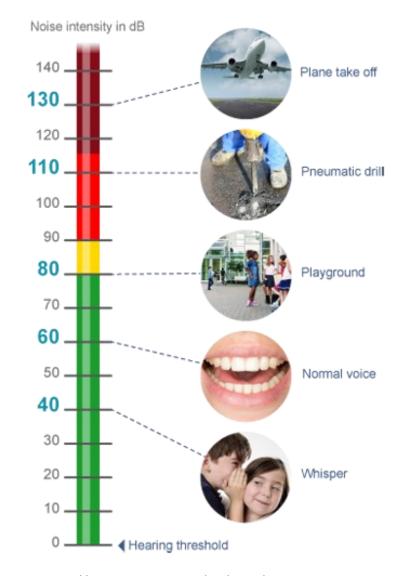


image source: https://www.cochlea.org/en/hear/human-auditory-range

References and Further Reading

harris, fred, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform", Proceedings of the IEEE, vol 66, No 1, Jan 1978

Enrico's Chart of Phase Noise and Two-Sample Variances: https://rubiola.org/pdf-static/Enrico's-chart-EFTS.pdf

Allan, David. Statistics of Atomic Frequency Standards, pages 221–230. Proceedings of the IEEE, Vol. 54, No 2, February 1966.

Anders Wallin, The allantools python library: https://allantools.readthedocs.io/en/latest/

P. Welch, "The use of the fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms", IEEE Trans. Audio Electroacoust. vol. 15, pp. 70-73, 1967.

Scipy.signal welch function: https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.welch.html

My StackExchange posts including detailing non-traditional use of ADEV for non-clock or gyroscope waveforms:

https://dsp.stackexchange.com/questions/82004/how-to-interpret-adev-plot/82005#82005

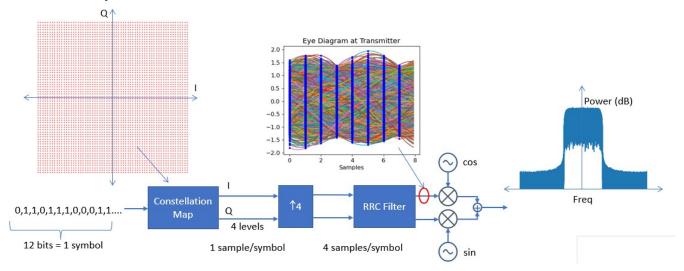
 $\frac{https://dsp.stackexchange.com/questions/88879/1-f-noise-why-does-the-allan-deviation-remain-constant-while-standard-error-of/88892\#88892$

https://dsp.stackexchange.com/questions/87466/usefulness-of-allan-deviation-with-dc-signals/87468#87468

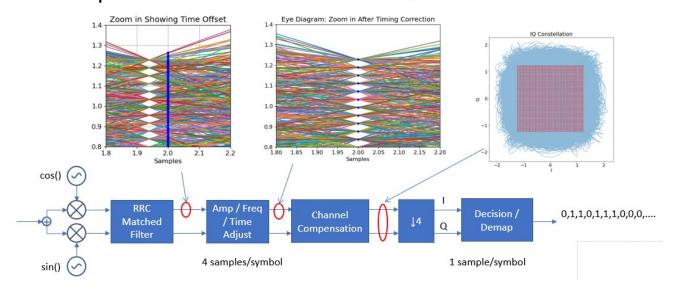
https://dsp.stackexchange.com/questions/83350/fft-to-work-out-optimum-number-of-samples-to-average/83354#83354

https://dsp.stackexchange.com/questions/87402/allan-deviation-to-determine-averaging-time/87403#87403

Example Waveform: 4096-QAM Transmitter



Example Waveform: 4096-QAM Receiver



Banner Graphic for Notebook

