Flinders University

WE ARE ON KAURNA LAND

Flinders University acknowledges the Traditional Owners and Custodians of the lands on which its campuses are located, these are the Traditional Lands of the Arrernte, Dagoman, First Nations of the South East, First Peoples of the River Murray & Mallee region, Jawoyn, Kaurna, Larrakia, Ngadjuri, Ngarrindjeri, Ramindjeri, Warumungu, Wardaman and Yolngu people. We honour their Elders past, present and emerging.

Today, over **400 ABORIGINAL AND TORRES STRAIT ISLANDER STUDENTS** are enrolled in courses at Flinders University.

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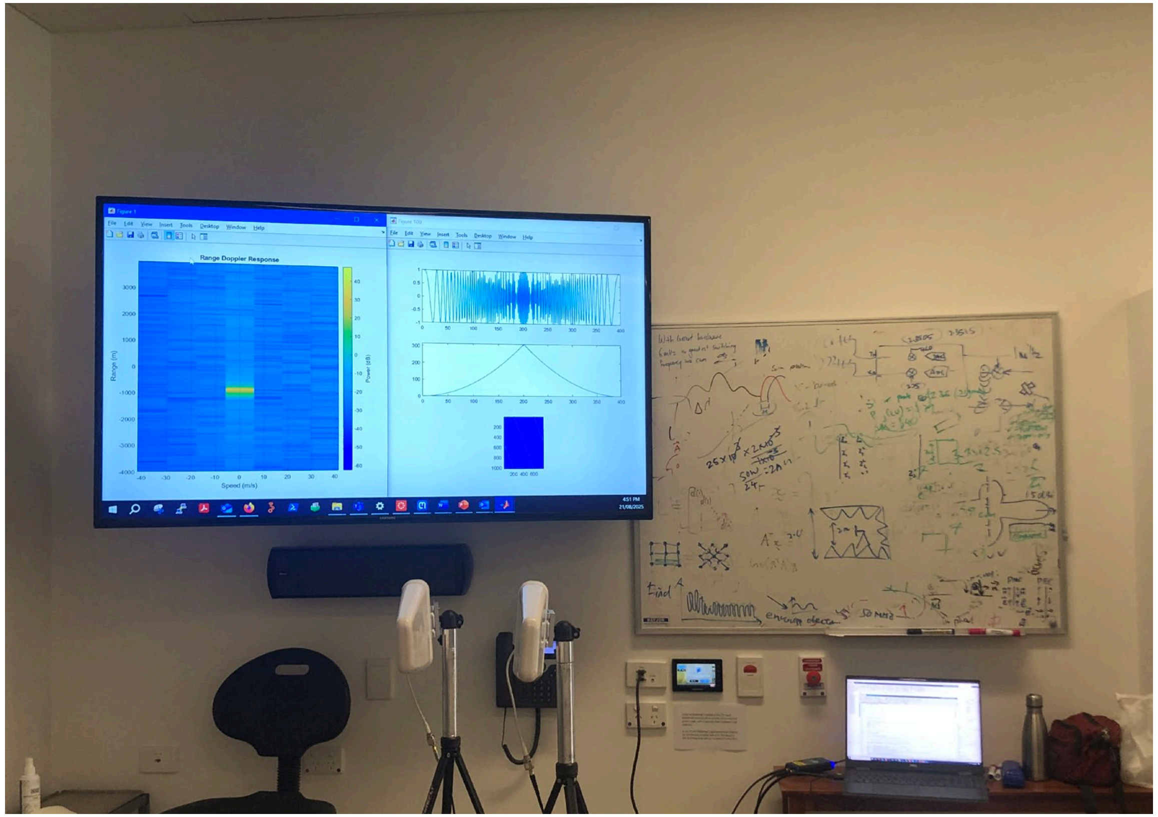
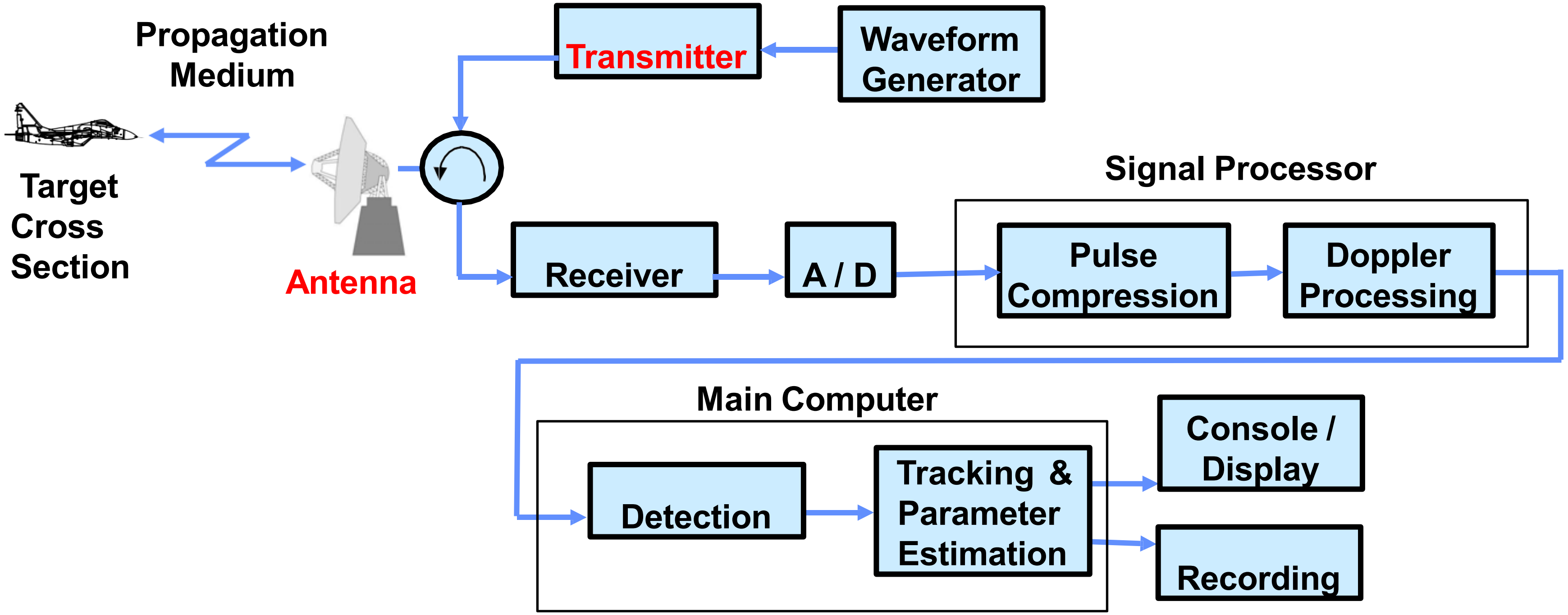
The Radar Equation



- Sourced from: https://www.ll.mit.edu/outreach/online-course-radar-introduction-radar-systems



The Radar Range Equation Connects:

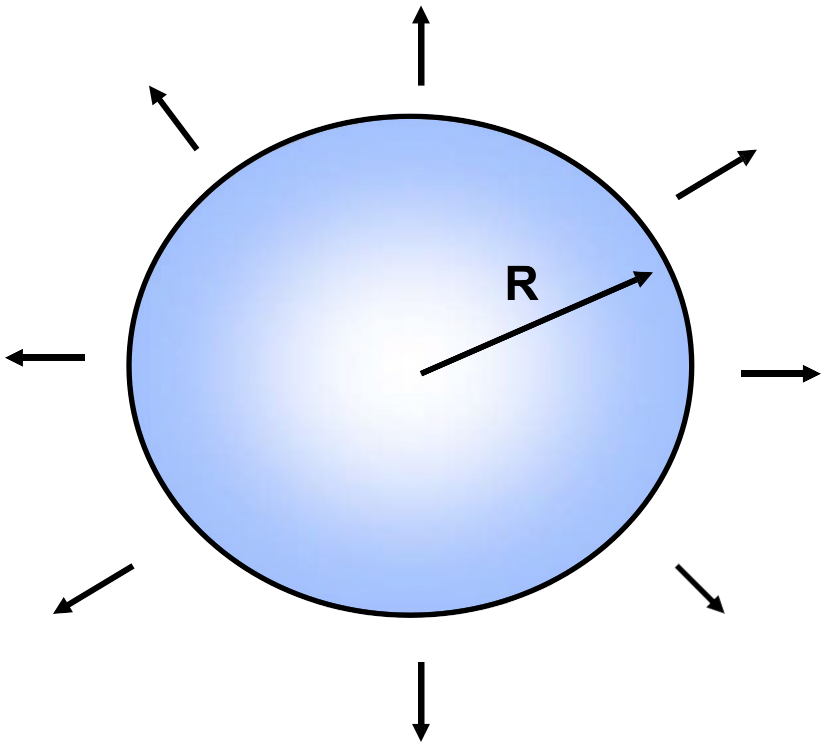
1. **Target Properties - e.g. Target Reflectivity (radar cross section)**
2. **Radar Characteristics - e.g. Transmitter Power, Antenna Aperture**
3. **Distance between Target and Radar - e.g. Range**
4. **Properties of the Medium - e.g. Atmospheric Attenuation.**

- **Introduction**
- **Introduction to Radar Equation**
- **Radar Losses**
- **Example**
- **Summary**

Power density from uniformly radiating antenna transmitting spherical wave

$$\frac{P_t}{4 \pi R^2}$$

P_t = peak transmitter power
R = distance from radar



Power density from isotropic antenna

$$\frac{P_t}{4 \pi R^2}$$

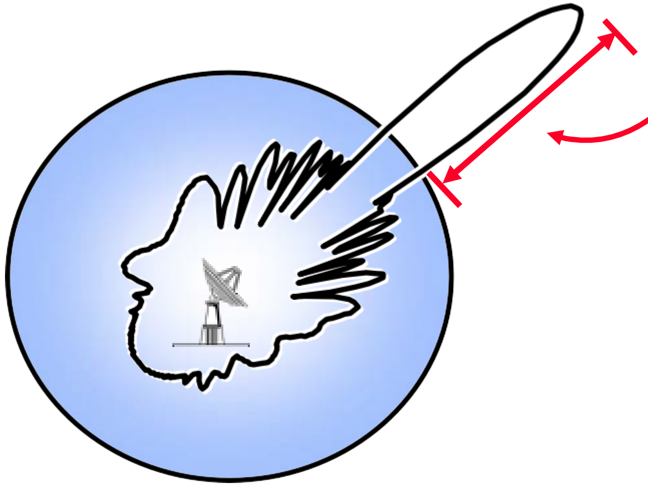
Power density from directive antenna

$$\frac{P_t G_t}{4 \pi R^2}$$

P_t = peak transmitter power

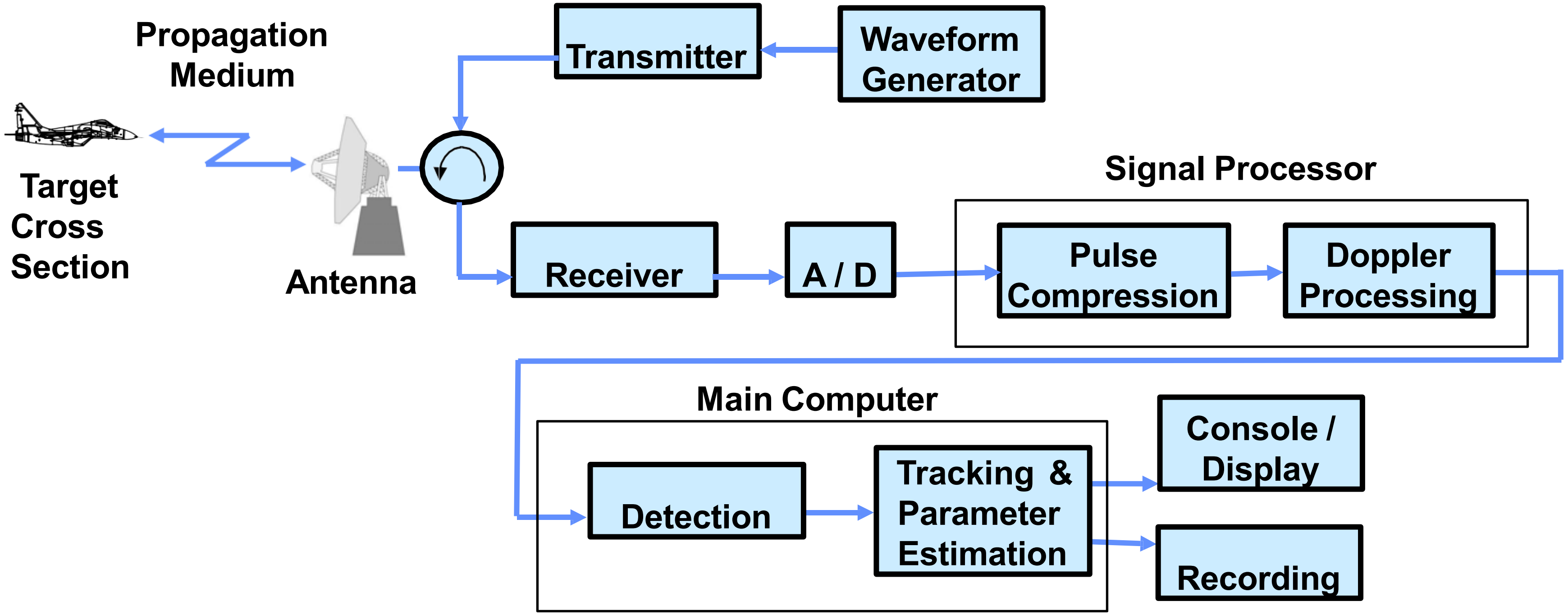
R = distance from radar

G_t = transmit gain



Gain is the radiation intensity of the antenna in a given direction over that of an isotropic (uniformly radiating) source

$$\text{Gain} = 4 \pi A / \lambda^2$$



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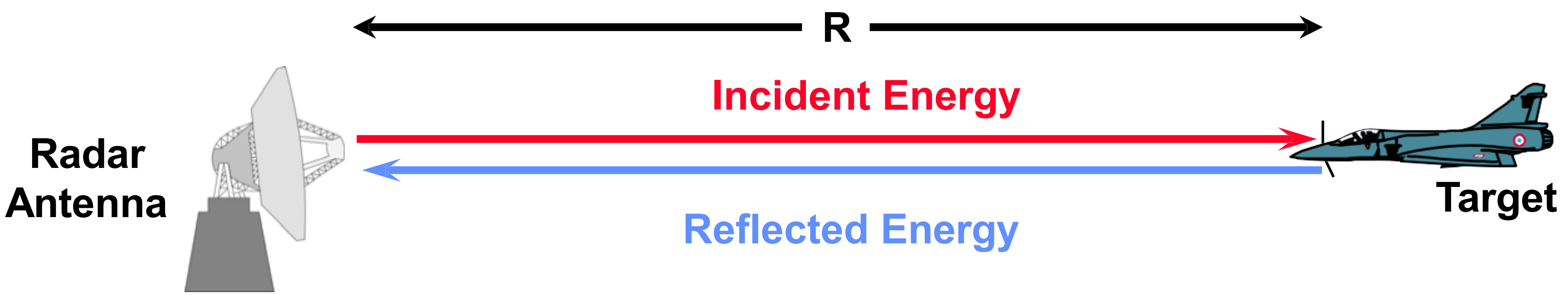

Radar Cross Section (RCS or σ) is a measure of the energy that a radar target intercepts and scatters back toward the radar

Power of reflected signal at target $\quad \dfrac{P_t G_t \sigma}{4 \pi R^2}$

Power density of reflected signal at the radar $\quad \dfrac{P_t G_t}{4 \pi R^2} \dfrac{\sigma}{4 \pi R^2}$

$\sigma =$ radar cross section units (meters)^2

Power density of reflected signal falls off as ($1/R^2$)

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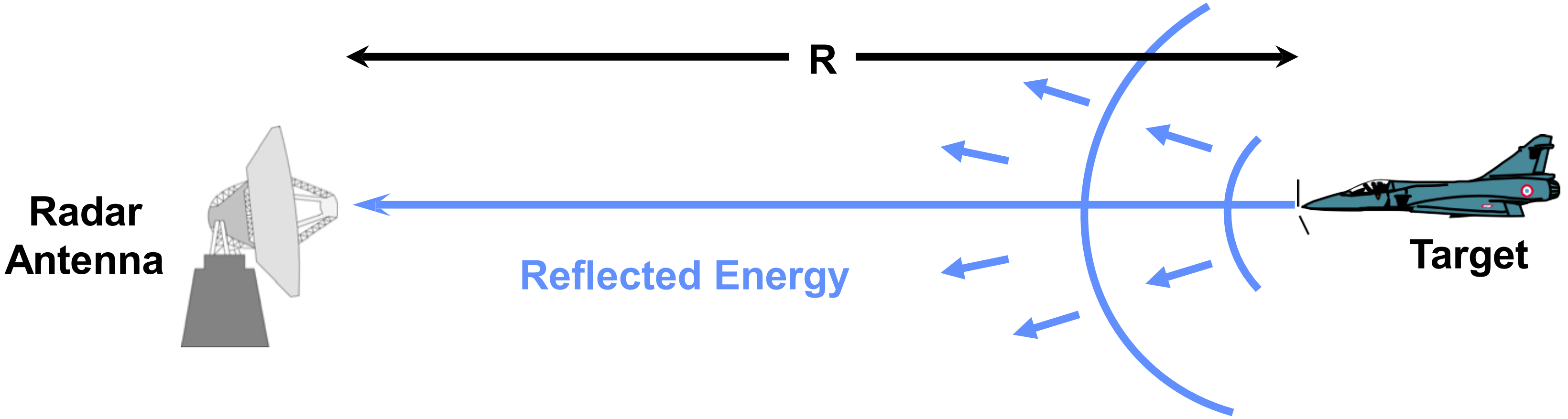
Power density of reflected signal at radar

$$\frac{P_t G_t}{4 \pi R^2} \; \frac{\sigma}{4 \pi R^2}$$



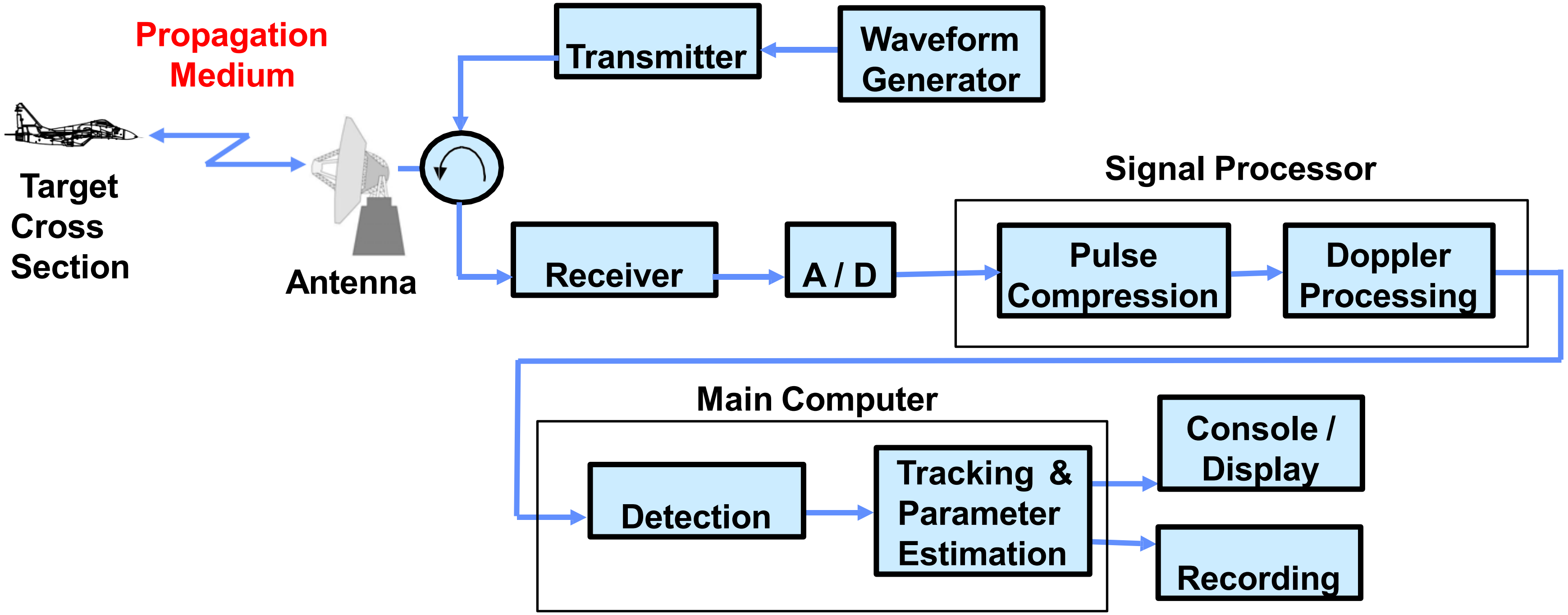
The received power = the power density at the radar times the area of the receiving antenna

Power of reflected signal from target and received by radar

$$P_r = \frac{P_t G_t}{4 \pi R^2} \; \frac{\sigma A_e}{4 \pi R^2}$$

P_r = power received

A_e = effective area of receiving antenna



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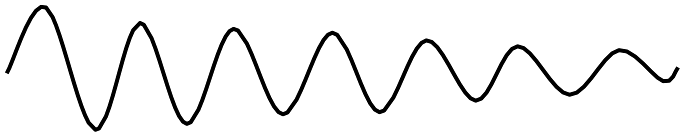
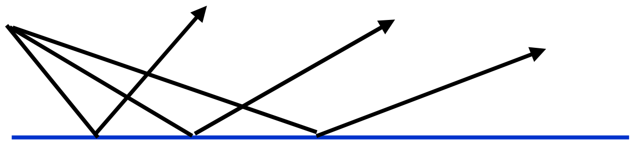
Propagation Effects on Radar Performance

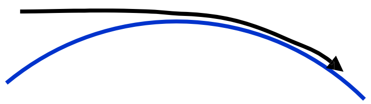
- **Atmospheric attenuation**



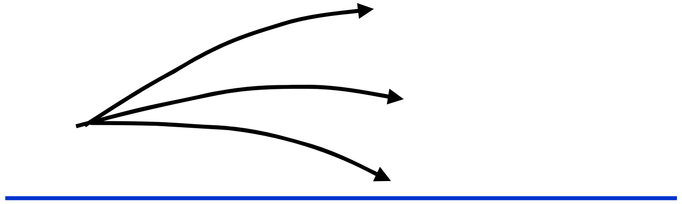
- **Reflection off of Earth’s surface**



- **Over-the-horizon diffraction**



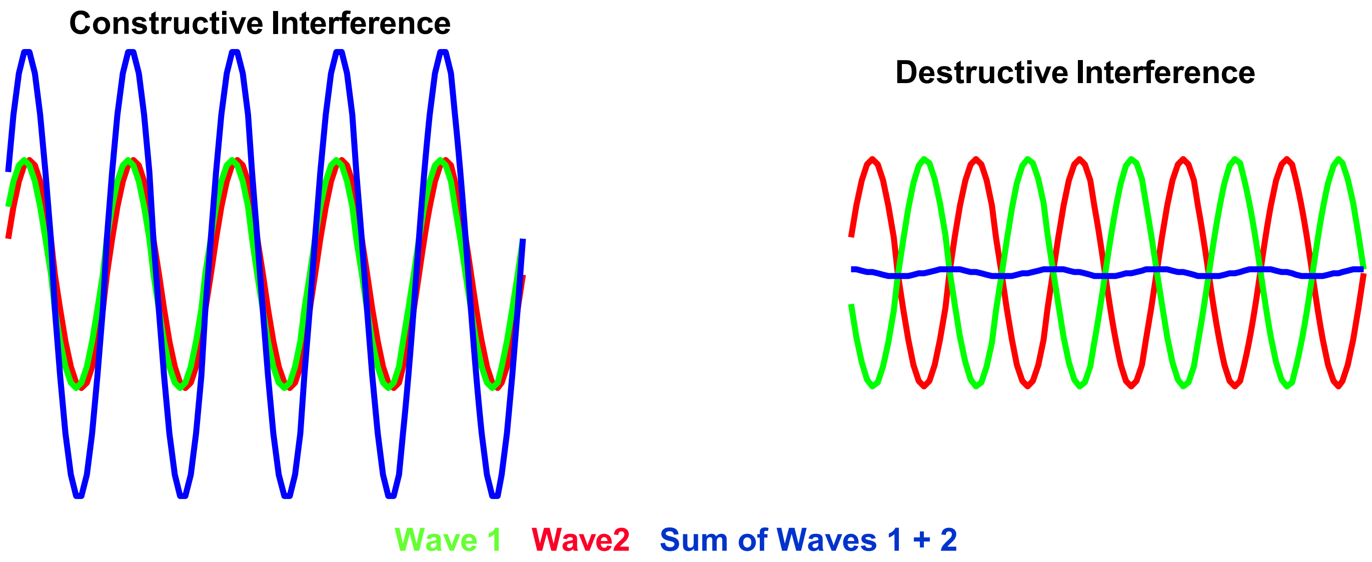
- **Atmospheric refraction**



Radar beams can be attenuated, reflected and bent by the environment

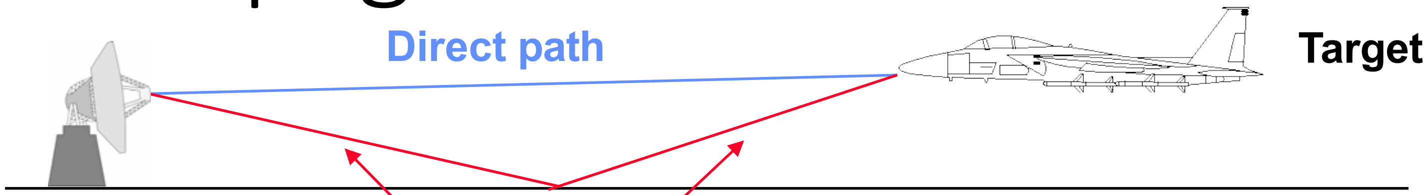
- **Atmospheric attenuation**
- **Reflection from the Earth's surface**
- **Over-the-horizon diffraction**
- **Atmospheric refraction**

Interference Basics (Channel Fading)



- **Two waves can interfere constructively or destructively**
- **Resulting field strength depends only on relative amplitude and phase of the two waves**
  - **Radar voltage can range from 0-2 times single wave**
  - **Radar power is proportional to (voltage)^2 for 0-4 times the power**
  - **Interference operates both on outbound and return trips for 0-16 times the power**

Propagation over a Plane Earth



Reflection from the Earth's surface results in interference of the direct radar signal with the signal reflected off of the surface

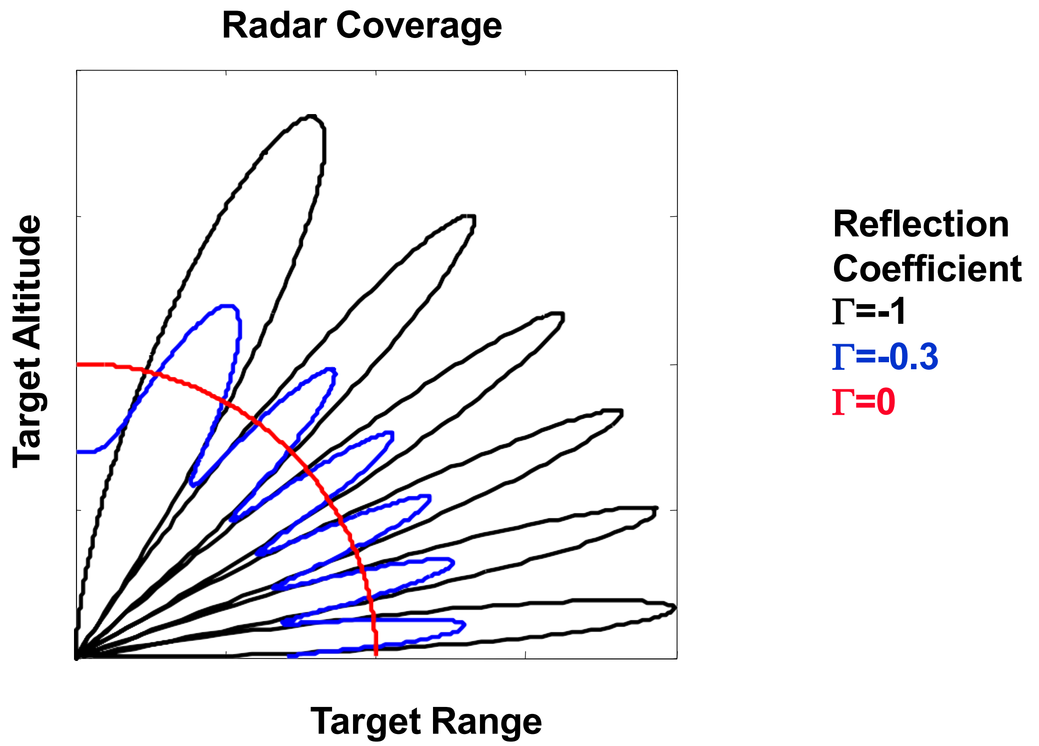
Surface reflection coefficient (Γ) determines relative signal amplitudes

- **Dependent on: surface material, roughness, polarization, frequency**
- **Close to 1 for smooth ocean, close to 0 for rough land**

Relative phase determined by path length difference and phase shift on reflection

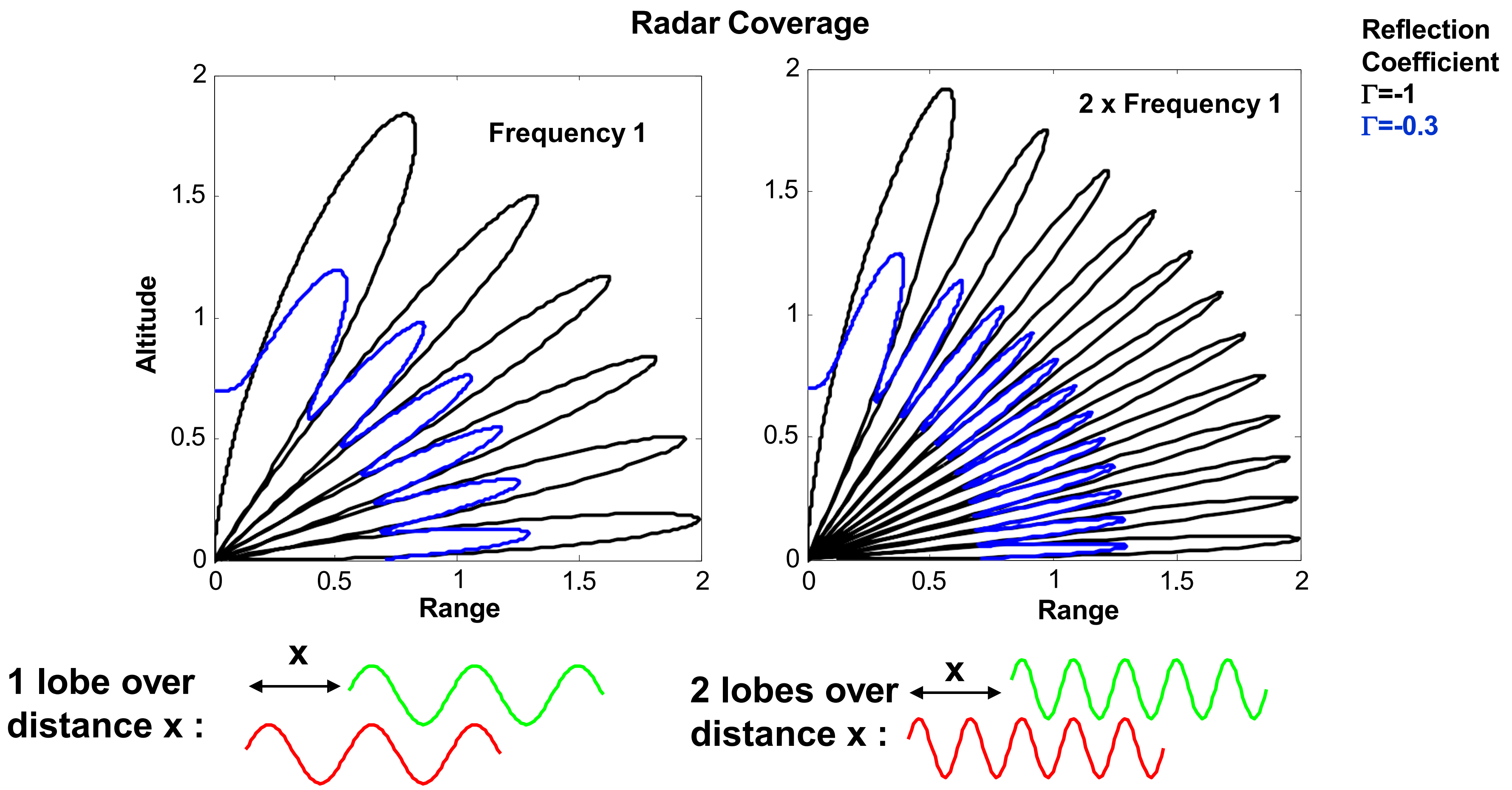
- **Dependent on: height, range and frequency**

Multipath Alters Radar Detection Range



- **Multipath causes elevation coverage to be broken up into a lobed structure**
- **A target located at the maximum of a lobe will be detected as far as twice the free-space detection range**
- **At other angles the detection range will be less than free space and in a null no echo signal will be received**

Multipath is Frequency Dependent

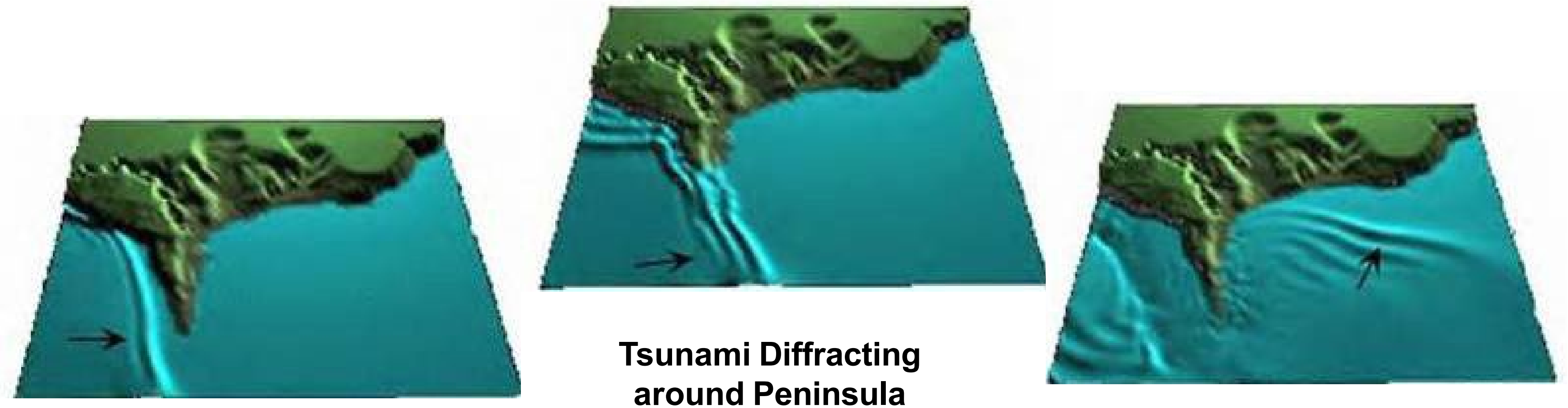


Lobing density increases with increased radar frequency

Outline

- **Atmospheric attenuation**
- **Reflection from the Earth's surface**
- **Over-the-horizon diffraction**
- **Atmospheric refraction**

Diffraction (Seeing Around Corners)

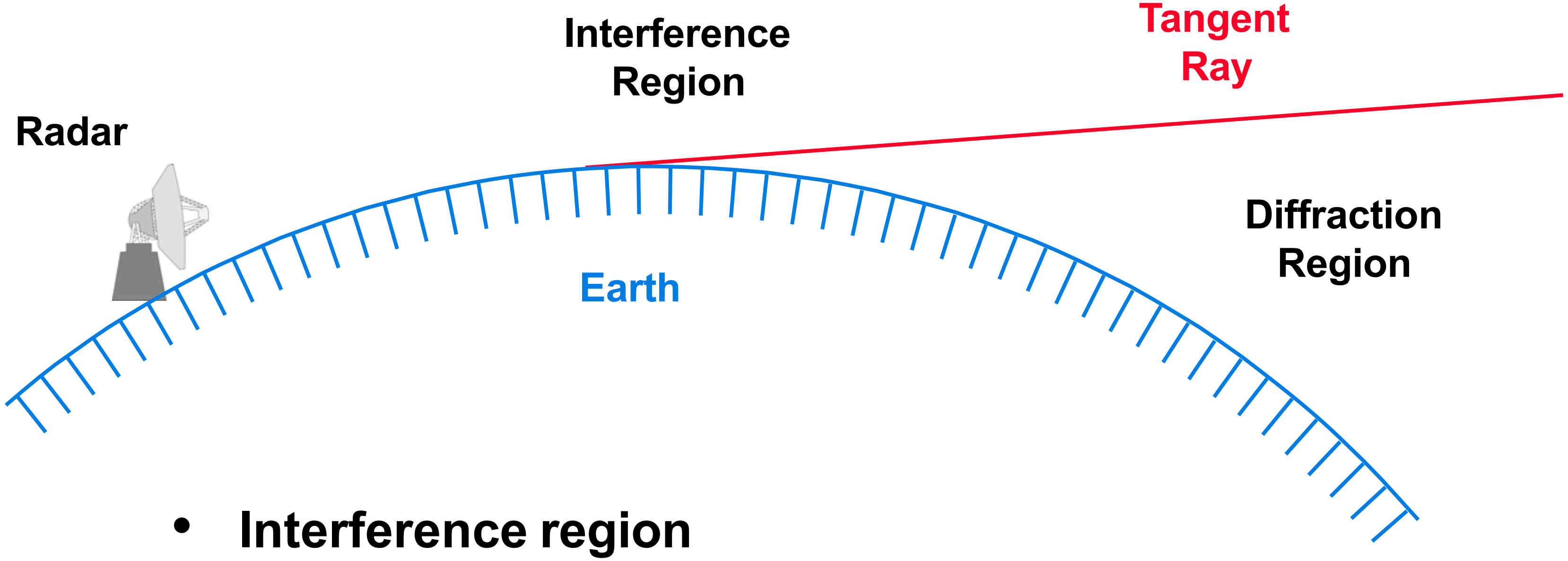


Tsunami Diffracting around Peninsula

Courtesy of NOAA / PMEL / Center for Tsunami Research.
See animation at http://nctr.pmel.noaa.gov/animations/Aonae.all.mpg

- **Radar waves are diffracted around the curved Earth just as ocean waves are bent by an obstacle**
- **Web references for excellent water wave photographic examples:**
  - **http://upload.wikimedia.org/wikipedia/commons/b/b5/Water_diffraction.jpg**
  - **http://yhspatriot.yorktown.arlington.k12.va.us/~ckaldahl/wave.gif**
- **The ability of radar to propagate beyond the horizon depends upon frequency and radar height**

Propagation Over Round Earth



- **Interference region**
  - **Located within line-of-sight radar**
- **Diffraction region**
  - **Below radar line of sight**
  - **Signals are severely attenuated**

Outline

- **Atmospheric attenuation**
- **Reflection from the Earth's surface**
- **Over-the-horizon diffraction**
- **Atmospheric refraction**

Refraction of Radar Beams

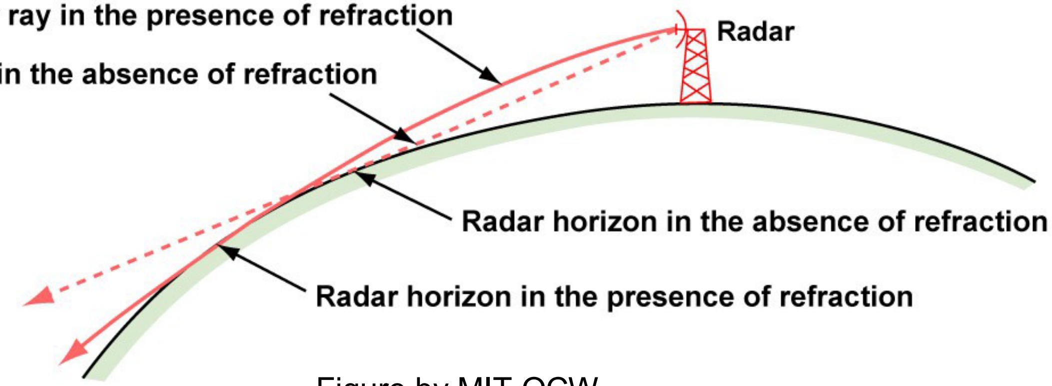
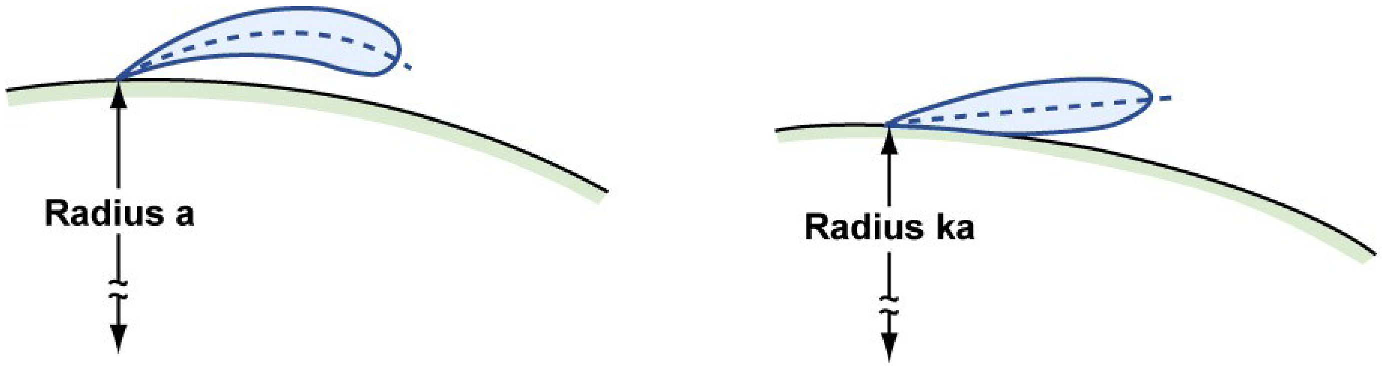


Figure by MIT OCW.

Radar rays bend downwards due to decreasing index of refraction of air with altitude

Same effect as refraction of light beam shining from water into air

Earth's Radius Modified to Account for Refraction Effects



Antenna beam bent due to refraction by the Earth's atmosphere

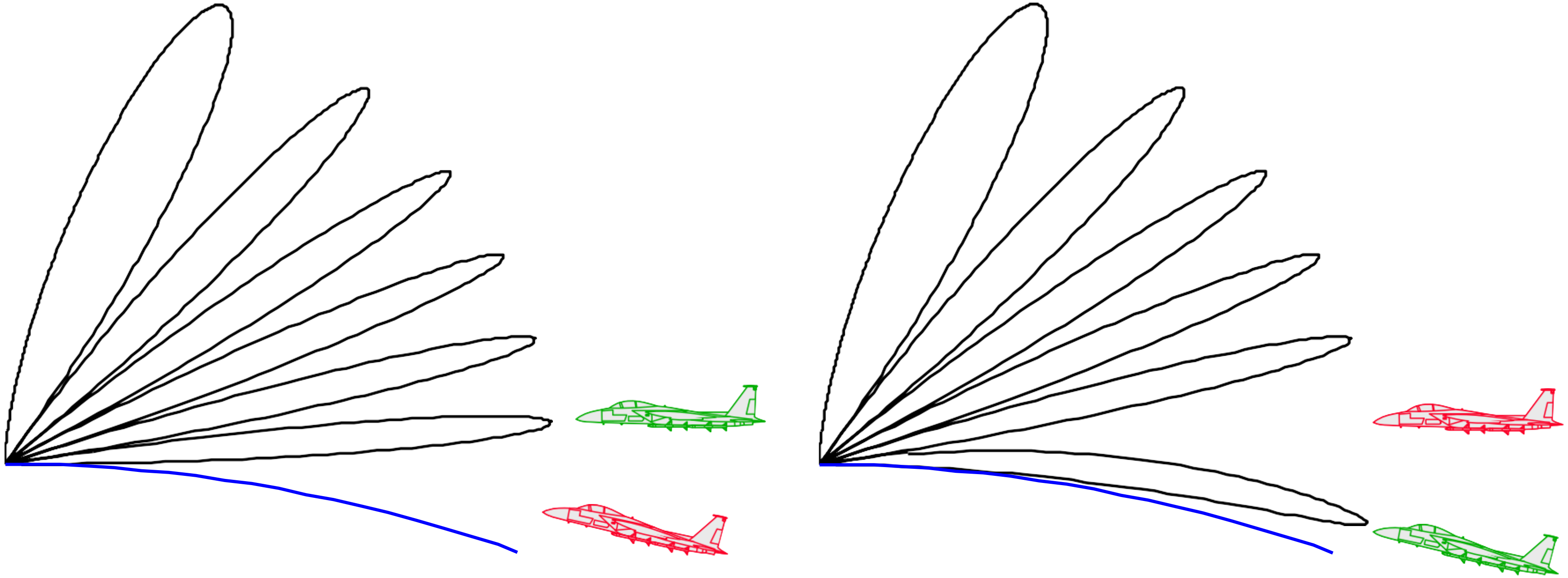
Shape of beam in equivalent Earth representation with radius ka

Atmospheric refraction is accounted for by replacing the actual Earth radius a, in calculations, by an equivalent earth radius ka and assuming straight line propagation

4/3 is a typical value for k

Average propagation is referred to as a "4/3 Earth"

Ducting Effects on Target Detection

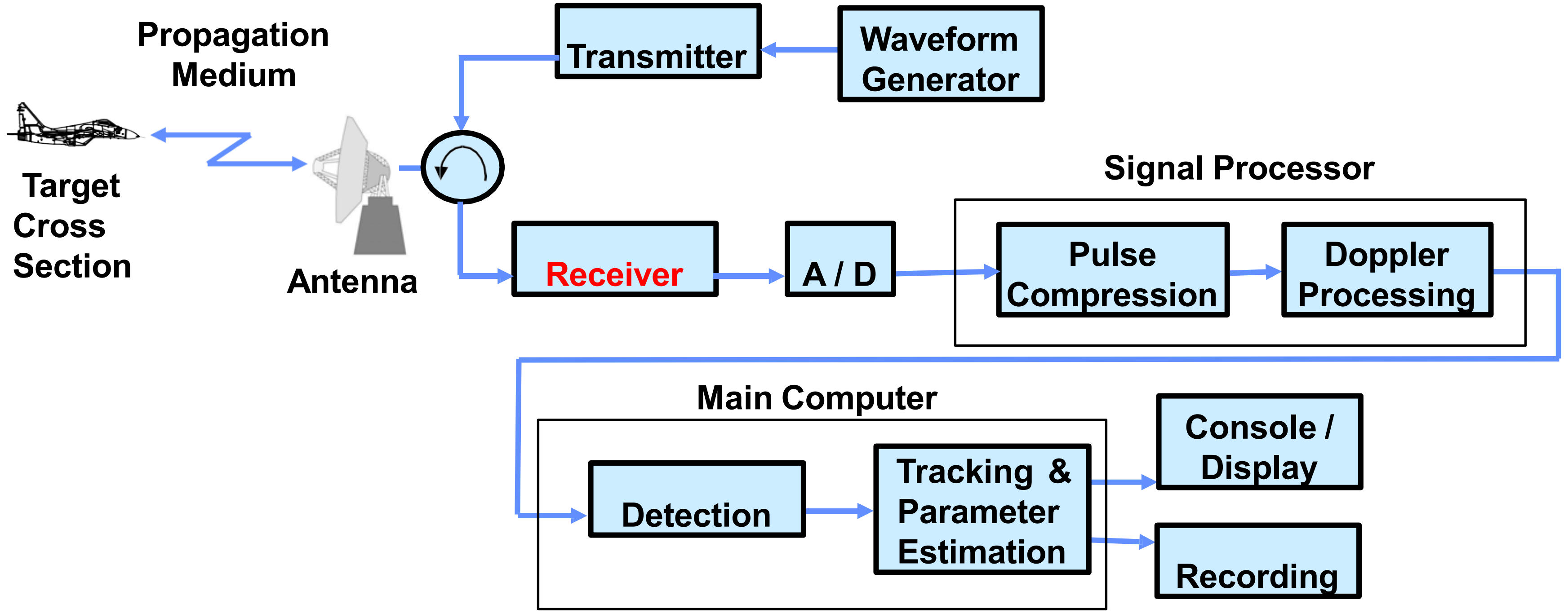


No Surface Duct

Surface Duct

Target seen
Target not seen

Ducting extends low-altitude detection ranges but can cause unexpected holes in radar coverage



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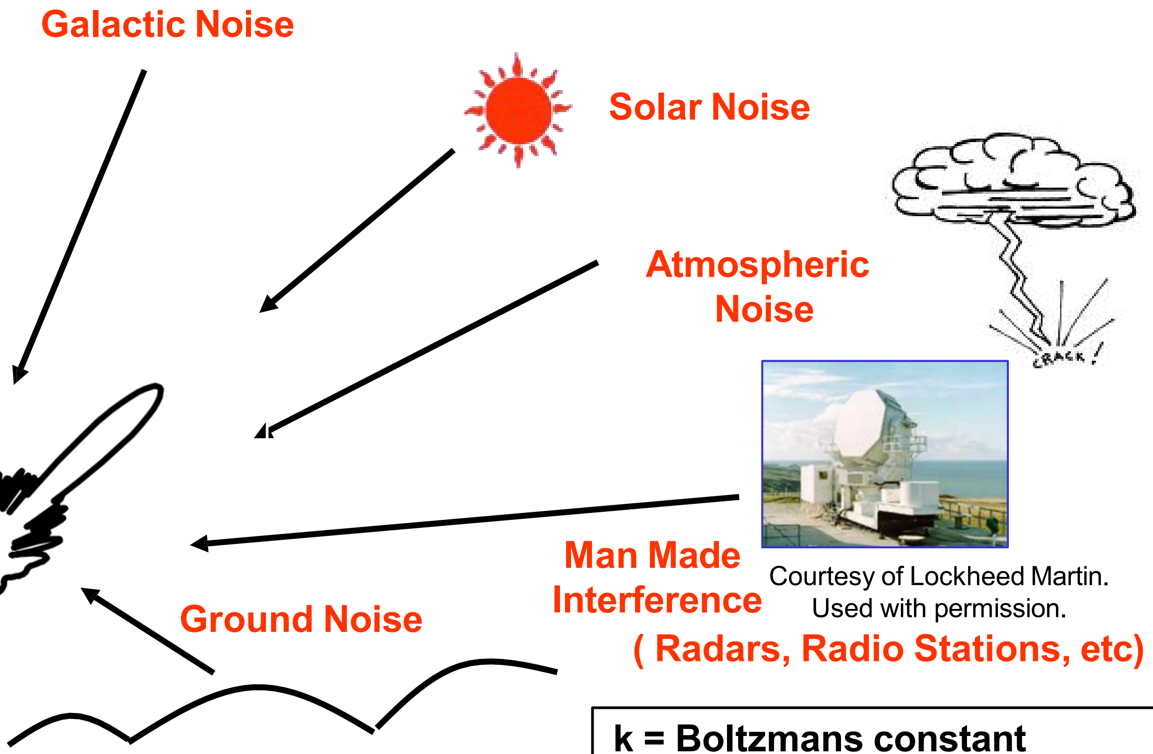
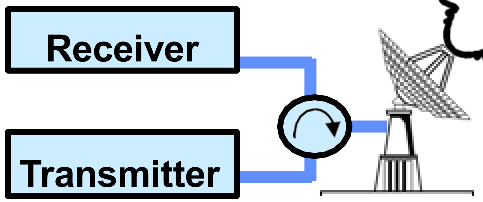
Noise from Many Sources Competes with the Target Echo

- The total effect of these noise sources is represented by a single noise source at the antenna output terminal.

- The noise power at the receiver is given by:

$N = k\,B_n\,T_s$





(Receiver, waveguide, and duplexer noise)

k = Boltzmans constant
= 1.38×10^{-23} joules / deg ^{o}K
T_s= System Noise Temperature
B_n = Noise bandwidth of receiver

Signal Power reflected from target and received by radar

$$P_r = \frac{P_t G_t}{4 \pi R^2} \frac{\sigma A_e}{4 \pi R^2}$$

Average Noise Power

$$N = k T_s B_n$$

Signal to Noise Ratio

$$S / N = P_r / N$$

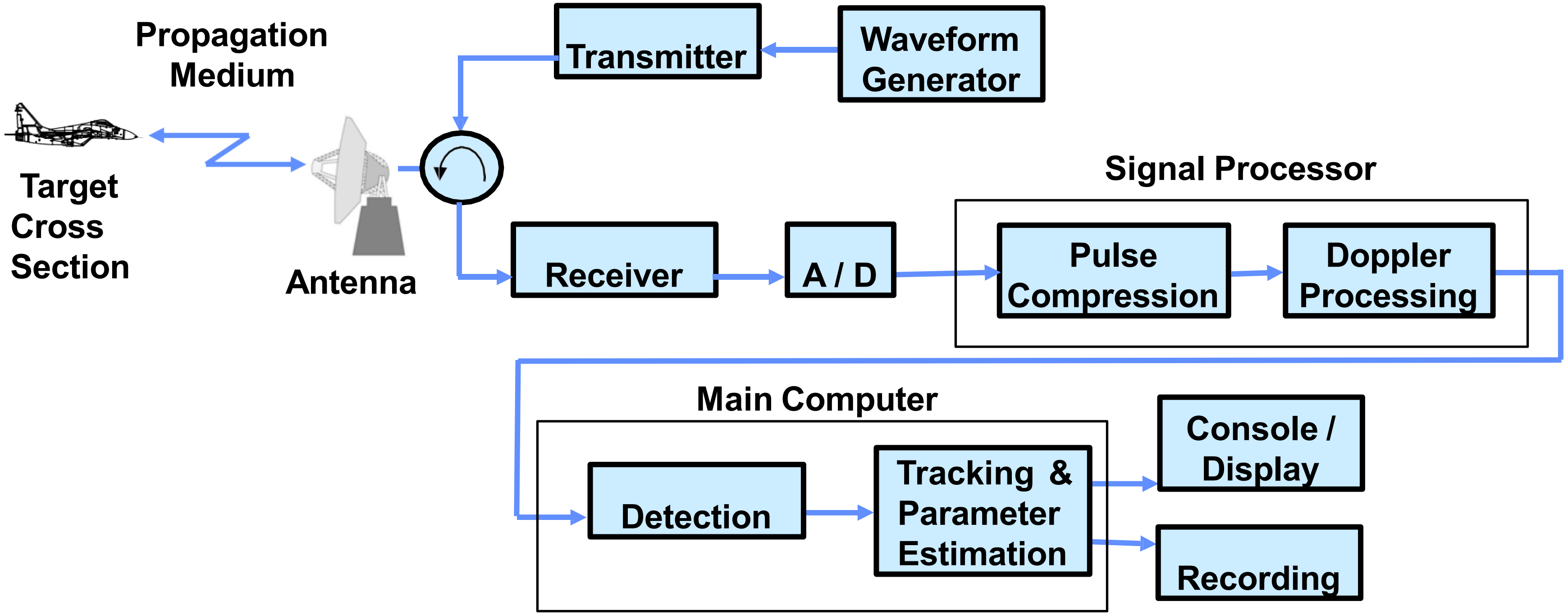
Assumptions :
$G_t = G_r$
L = Total System Losses
$T_o = 290^o K$

$$S / N = \frac{P_t G^2 \lambda^2 \sigma}{(4 \pi)^3 R^4 k T_s B_n L}$$

Signal to Noise Ratio (S/N or SNR) is the standard measure of a radar's ability to detect a given target at a given range from the radar

" S/N = 13 dB on a 1 m² target at a range of 1000 km"

radar cross section of target



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- **The System Noise Temperature, T_S , is divided into 3 components :**

$$T_s = T_a + T_r + L_r T_e$$

- **T_a is the contribution from the antenna**
  - **Apparent temperature of sky (from graph)**
  - **Loss within antenna**
- **T_r is the contribution from the RF components between the antenna and the receiver**
  - **Temperature of RF components**
- **L_r is the loss of input RF components**
- **T_e is the temperature of the receiver**
  - **Noise factor of receiver**

- Introduction
- Introduction to Radar Equqtion
- Radar Losses
- Example
- Summary

- **Problem : Show that a radar with the parameters listed below, will get a reasonable S / N on an small aircraft at 60 nmi.**

Radar Parameters

Range	60 nmi
Aircraft cross section	1 m^2
Peak Power	1.4 Megawatts
Duty Cycle	0.000525
Pulsewidth	.6 microseconds
Bandwidth	1.67 MHz
Frequency	2800 MHz
Antenna Rotation Rare	12.8 RPM
Pulse Repetition Rate	1200 Hz
Antenna Size	4.9 m wide by 2.7 m high
Azimuth Beamwidth	1.35 ^{o}
System Noise Temp.	950 ^{o} K

$\lambda =$ c / f = .103 m

G = 4 π A / λ^2 = 15670 m^2
= 42 dB, (actually 33 dB with beam shaping losses)

Number of pulses per beamwidth = 21

Assume Losses = 8dB

$$S / N = \frac{P_t \, G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k \, T_s \, B_n \, L}$$

P_t = 1.4 Megawatts	R = 111, 000 m
G = 33 dB = 2000	T_s = 950 °K
$\lambda = .1$ m	B_n = 1.67 MHz
$\sigma = 1$ m^2	L = 8dB = 6.3
k = 1.38 x 10^{-23} w / Hz ° K	$(4\pi)^3$ = 1984

$$\frac{(1.4 \times 10^6 \text{ w})(2000)(2000)(.1\text{m})(.1\text{m})(1\text{m}^2)}{(1984)\,(1.11 \text{ X } 10^5 \text{ m})^4\,(1.38 \times 10^{-23} \text{ w / Hz} \circ \text{K})\,(950 \circ \text{K})\,(6.3)\,(1.67 \times 10^6 \text{ Hz})}$$

$$\frac{5.6 \times 10^{+6+3+3-1-1}}{415 \times 10^{+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+2+3+20-23+2+6}} = \frac{5.6 \times 10^{+10}}{4.15 \times 10^{+10}} = 1.35 = 1.3 \text{ dB}$$

S / N = 1.3 dB per pulse (21 pulses integrated) => S / N per dwell = 14.5 dB

+ 13.2 dB

dB Method

		(+)	(-)
Peak Power	1.4 MW	61.5	
(Gain) ^{2}	33 db	66	
(Wavelength) ^{2}	.1 m		20
Cross section	1 m^2	0	
$(4 \pi)^3$	1984		33
(Range)^4	111 km		201.8
k	1.38 x 10 $^{-23}$ w / Hz ^{o} K	228.6	
System temp	950		29.8
Losses	8 dB		8
Bandwidth	1.67 MHz		62.2
		+ 356.1	- 354.8

+ 1.3 dB

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The Sanity Check

Take a Candidate Radar Equation

$$\frac{P A^2}{\lambda^2 k T_s L} = \frac{4 \pi R^4 (S/N)}{\sigma \tau}$$

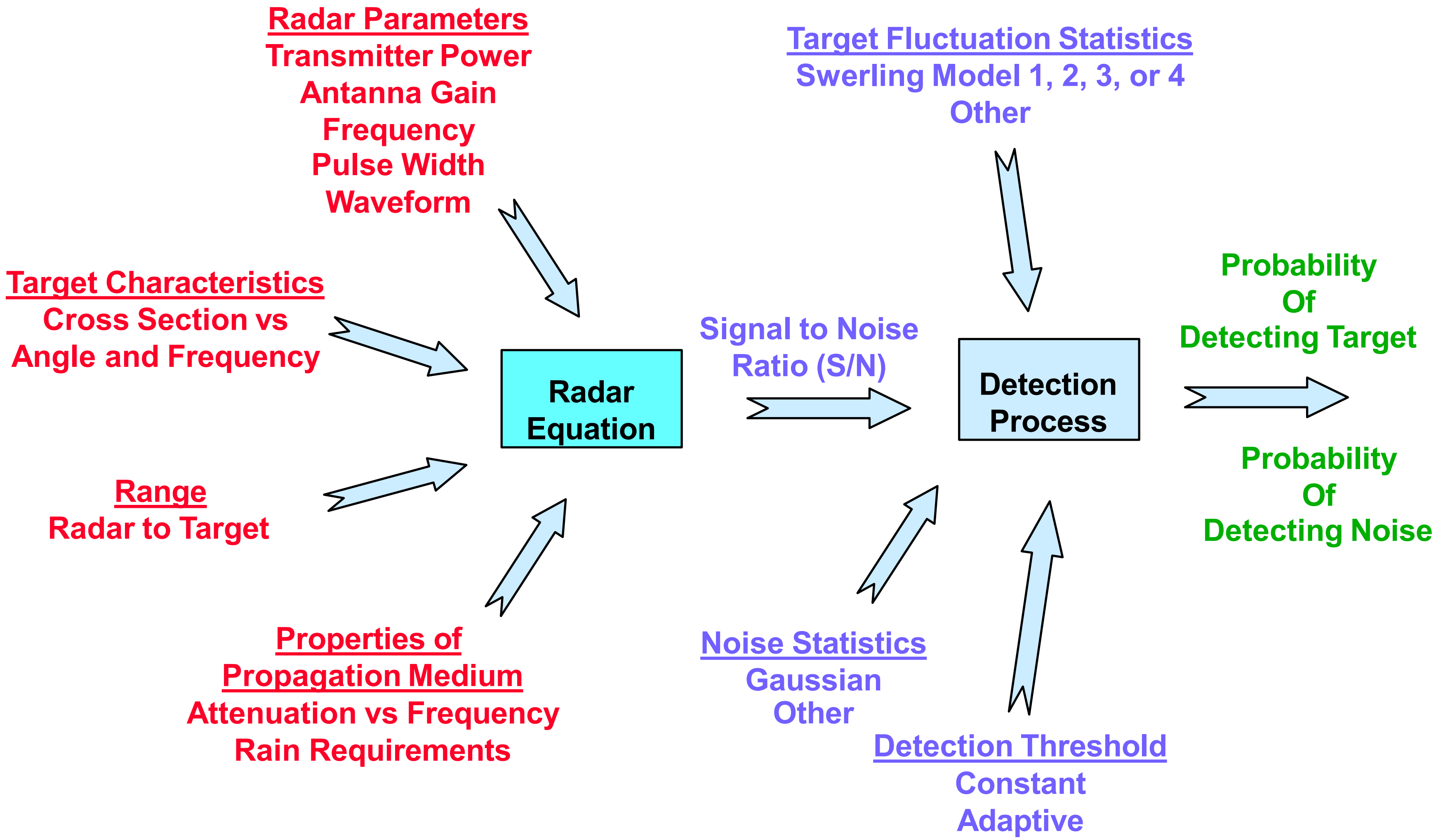
Check it Dimensionally

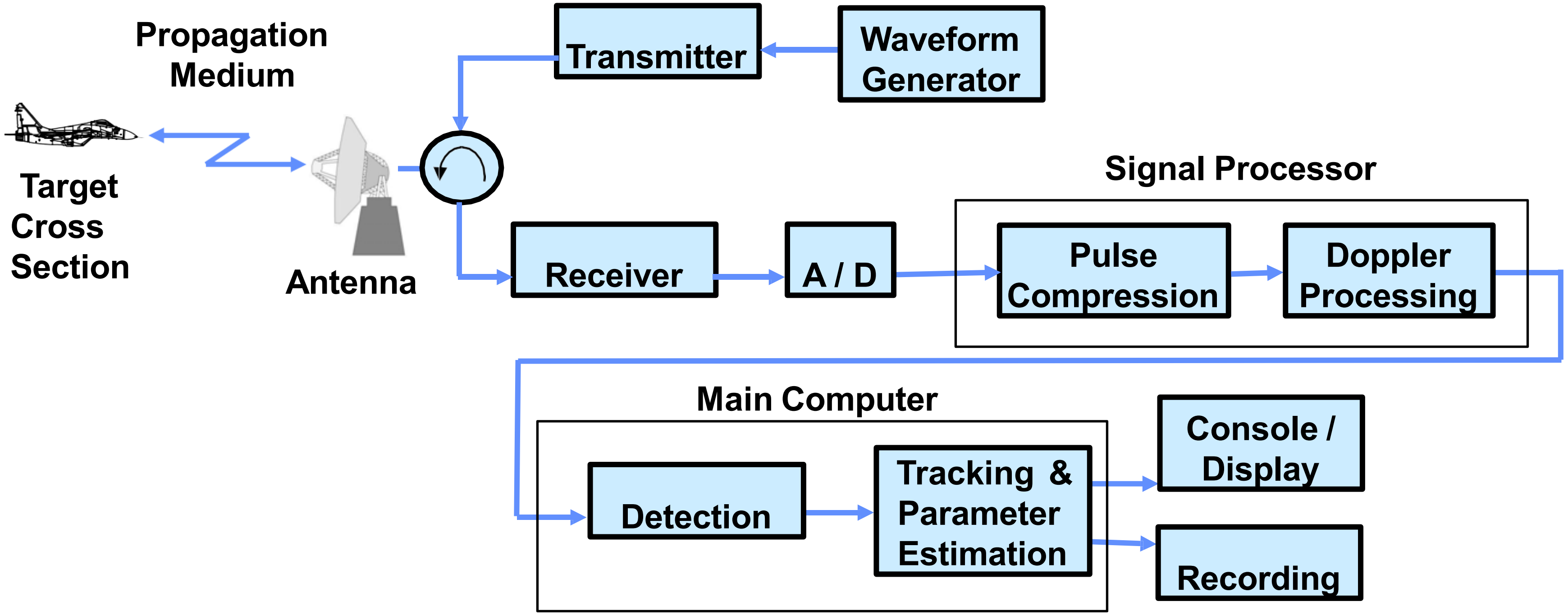
- P is energy/time
- kT_s is energy
- A and σ are distance squared
- λ and R are distance
- τ is time
- S/N, L and 4π are dimensionless

Check if Dependencies Make Sense

– Increasing Range and S/N make requirements tougher
– Decreasing σ and τ makes requirements tougher
– Increasing P and A make radar more capable
– Decreasing Noise Temp and Loss make radar more capable
– Decreasing λ makes radar more capable

Radar Equation and Detection Process





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