New Methods for FM Threshold Extension in GNU Radio

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Background

- FM Waveform

$$x(t) = A_c \; cos(2\pi f_c t + \int_0^t m(\tau) \; d\tau) \tag{1}$$

- $m(t)$ is real, band-limited, message signal
- f_c is the carrier frequency (Hz)
- Constant envelope

- Single tone message

$$m(t) = 2\pi k_f \; cos(2\pi \; f_m t) \tag{2}$$

- f_m is the message frequency (Hz)
- k_f is the peak frequency deviation (Hz)
- Modulation index - rough measure of the *Gain* or *Bandwidth Expansion*

$$\beta = \frac{k_f}{f_m} \tag{3}$$

- Spectrum of the signal $x(t)$ will be centered around f_c
- Occupied bandwidth approximated by the well known Carson's rule:

$$W_c \approx 2(k_f + f_{max}) \tag{4}$$

  where f_{max} is the highest frequency component of $m(t)$. Reference [1] explains more about this rule.

- Complex baseband form:

$$\begin{aligned} x_{bb}(t) &= x_i(t) + jx_q(t) \\ &= A_c e^{j \int_0^t m(\tau)\ d\tau} \end{aligned} \tag{5}$$

- Demodulated estimate of $m(t)$ is

$$
\begin{aligned}
\hat{m}(t) &= \frac{d}{dt}\operatorname{Arg}(x_{bb}(t)) \\
&= \frac{d}{dt} Arctan\left(\frac{x_q(t)}{x_i(t)}\right) \\
&= \left(\frac{1}{1+\frac{x_q^2(t)}{x_i^2(t)}}\right)\frac{d}{dt}\left(\frac{x_q(t)}{x_i(t)}\right) \\
&= \frac{x_i(t)\frac{d}{dt}x_q(t) - x_q(t)\frac{d}{dt}x_i(t)}{x_i^2(t)+x_q^2(t)}
\end{aligned}
\tag{6}
$$

- This commonly approximated in SDR implementations.

Noise and the Threshold Effect

FM Demodulated SNR as a function of received CNR i

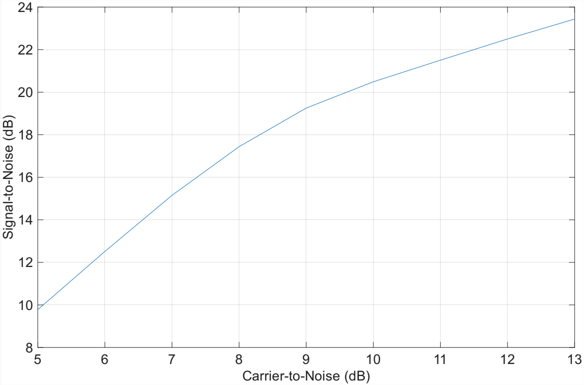

Figure 1: Example FM SNR Performance

FM Demodulated SNR as a function of received CNR ii

- Noise in the demodulated FM signal in [3] and [4]
- Based on work dating back to [2].
- At large CNR, the demodulated SNR of $\hat{m}(t)$ is approximated by:

$$\left(\frac{S_m}{N_m}\right) \approx \frac{3}{8}\left(\frac{S_c}{N_c}\right)\left(\frac{W_c}{W_m}\right)^3 \tag{7}$$

- $\frac{S_c}{N_c}$ is the CNR, W_c is the (two sided) bandwidth
- W_m is the bandwidth of $\hat{m}(t)$.

FM Click Event i

- Rice postulated that noise in the threshold region was due to click events
- Derived methods for estimating the probability of a click

FM Click Event ii

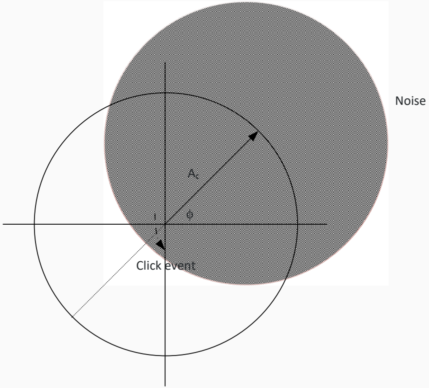

Figure 2: FM Click Event

Threshold Extension

FM Click Removal i

- Following [3] noise in the threshold region is due to click events
- Received points outside unit circle can be limited to unit circle (small angle error)
- Received points well inside the unit circle can have large angle errors (erase these)
- Interpolate the erased points from the remaining points
- Band limited interpolation over block of points with Papoulis-Gerchberg algorithm

FM Click Removal ii

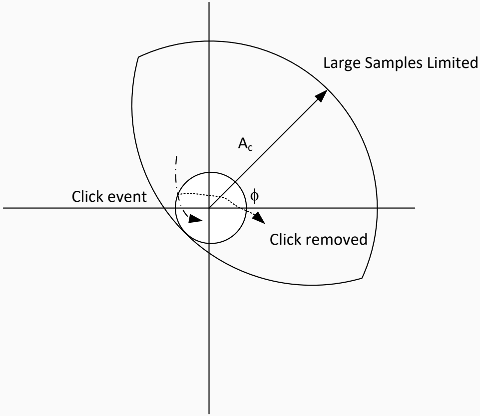

Figure 3: FM Click Removal

FM Click Removal iii

$$B = W^H \Gamma W \tag{8}$$

- W is the NxN discrete Fourier Transform (DFT) matrix
- Γ is a diagonal selector matrix to determine bandwidth of interpolation
- Set to match bandwidth from Carson's rule
- Multiple iterations until erased points are smooth

Simulation in GNU Radio

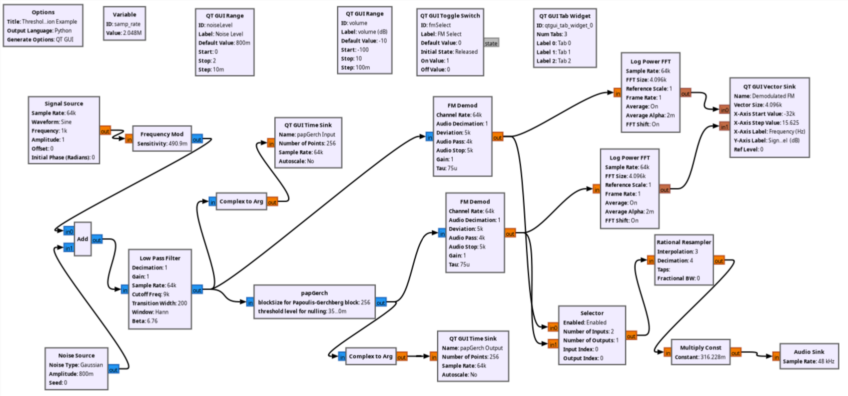

Figure 4: Flowgraph to Test Threshold Extension

Simulation Results ii

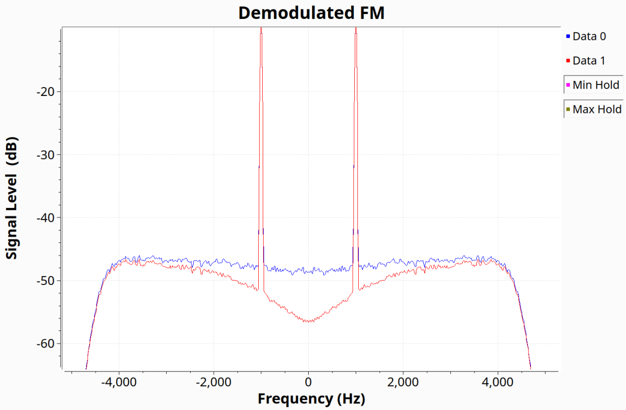

Figure 5: Example of Threshold Extension Improvement

Future Work

Future Work i

- FM project on https://github.com/W6COV/FM-Project
- Add parameters for blocksize, interpolation bandwidth, and erasure threshold
- Improve methods for identifying click events
- Analyze noise with order statistics

Bibliography

Bibliography i

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