New Methods for FM Threshold Extension in GNU Radio

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Abstract

This work describes new techniques for low carrier-to-noise ratio (CNR) FM demodulation and their implementation in GNU Radio. FM radio was developed nearly 90 years ago and is still in common use for commercial broadcasting, emergency services, and amateur radio. They key advantage of FM over other analog modulations (i.e. AM and SSB) is the *threshold effect* where received signals with a CNR above a threshold level will be demodulated to audio signals with a signal-to-noise ratio (SNR) larger than the CNR. However, demodulated FM signals below the threshold are subject click noise, which rapidly degrades audio quality and intelligibility. With modern signal processing approaches, it is possible to build practical demodulators that reduce click noise, thus improving communication networks that employ FM.

The presentation begins with a review of the classical theory of noise in FM receivers based on the work of S.O. Rice(Rice, 1962). Next, it covers a new technique for FM demodulation that leverages the natural structure of FM signals in noise, including the treatment of large noise events as *erasures*. Once identified, these erasures can be removed using band-limited signal interpolation with the discrete Papoulis-Gershberg algorithm(Ferreira, 1994).

Last, an example implementation of the modern FM demodulator in GNU Radio is demonstrated and compared with commercial receivers for amateur radio communications.

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1. Background

In frequency modulation, the message signal, typically audio is mapped onto the phase of a sinewave. Traditionally, this is written as:

$$x(t) = A_c \, cos(2\pi f_c t + \int_0^t m(\tau) \, d\tau) \tag{1}$$

where $m(t)$ is the real, band-limited, message signal and f_c is the carrier frequency in Hz. For a single tone message, set

$$m(t) = 2\pi k_f \, cos(2\pi \, f_m t) \tag{2}$$

where f_m is the message frequency (Hz) and k_f is the peak frequency deviation (Hz) of the carrier frequency. The modulation index is then defined as the ratio of the peak frequency deviation to the modulating frequency:

$$\beta = \frac{k_f}{f_m} \tag{3}$$

The (significant) frequency domain spectrum of the signal $x(t)$ will be centered around f_c and have an occupied bandwidth that can be approximated by the well known Carson's rule:

$$W_c \approx 2(k_f + f_{max}) \tag{4}$$

where f_{max} is the highest frequency component of $m(t)$. Unfortunately, this approximation underestimates the bandwidth in many practical systems (Carlson et al., 2002) but it is still useful for predicting the total signal bandwidth of the FM system.

To demodulate $x(t)$ and recover $m(t)$ it is simplest to shift the received signal to the complex baseband form (mathematically via Hilbert transforms or practically with an I/Q demodulator):

$$\begin{aligned} x_{bb}(t) &= x_i(t) + j x_q(t) \\ &= A_c e^{j \int_0^t m(\tau) \, d\tau} \end{aligned} \tag{5}$$

then the demodulated estimate of $m(t)$ is

$$\begin{aligned}\hat{m}(t) &= \frac{d}{dt}\,\mathrm{Arg}(x_{bb}(t)) \\ &= \frac{d}{dt} Arctan\left(\frac{x_q(t)}{x_i(t)}\right) \\ &= \left(\frac{1}{1+\frac{x_q^2(t)}{x_i^2(t)}}\right)\frac{d}{dt}\left(\frac{x_q(t)}{x_i(t)}\right) \\ &= \frac{x_i(t)\frac{d}{dt}x_q(t) - x_q(t)\frac{d}{dt}x_i(t)}{x_i^2(t)+x_q^2(t)}\end{aligned} \tag{6}$$

The final form is commonly approximated in SDR implementations.

2. Noise and the Threshold Effect

The noise in the demodulated FM signal is reviewed in (Rice, 1962) and (Taub & Schilling, 1986) based on work dating back to (Crosby, 1937). At large carrier-to-noise levels, the signal-to-noise ratio (SNR) of $\hat{m}(t)$ is approximated by:

$$\left(\frac{S_m}{N_m}\right) \approx \frac{3}{8}\left(\frac{S_c}{N_c}\right)\left(\frac{W_c}{W_m}\right)^3 \tag{7}$$

where $\frac{S_c}{N_c}$ is the carrier-to-noise ratio (CNR), W_c is the (two sided) bandwidth of the FM signal and W_m is the (one sided) bandwidth of $\hat{m}(t)$. An example is plotted below in Figure 1 for a $f_m = 1$ KHz sine wave modulated with a peak deviation of $k_f = 5$ KHz.

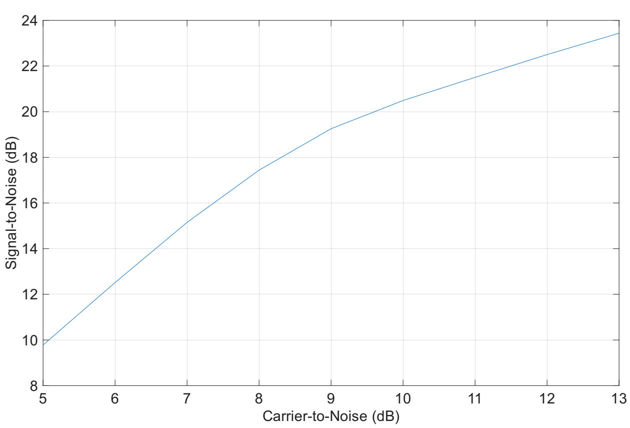

Figure 1. Example FM SNR Performance

The figure shows a threshold at an input carrier-to-noise level of about 9 dB. Above this level, the SNR has a gain of over the CNR approximately $10\ log_{10}\left(\frac{3}{8}\left(\frac{W_c}{W_m}\right)^3\right) =$ 10.5 dB. Below 9 dB CNR, however, the SNR degrades more quickly.

(Rice, 1962) developed a theory of click noise to explain the threshold effect and the degradation in SNR below the threshold. The key observation in his theory is that large amplitude noise can lead to phase errors in the received signal that wrap around the origin. Thus, the demodulator sees a spurious phase rotation of 2π radians. The concept is shown in Figure 2 below.

Here, the transmitted signal at a time t_0 is represent by a vector $x_{bb}(t_0) = A_c e^{j\phi}$. Complex Gaussian noise is added to the vector, resulting in received sequence in the gray region that can cross over the x-axis to the left of the origin. The measured differential of the phase angle along this path will have an erroneous impulse or *spike* of 2π. This results in broadband noise that has an audio click sound.

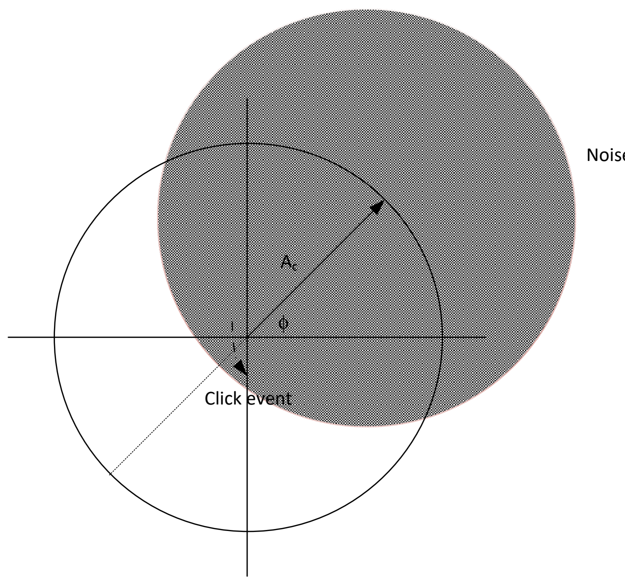

Figure 2. FM Click Event

As the CNR is decreased, the frequency of clicks increases as does their overall contribution to the demodulated noise.

3. Threshold Extension

3.1. Concepts

The goal of threshold extension is to improve FM demodulator performance at lower CNR levels in an attempt to retain the gain that is seen at high CNR. This work examines a method for processing input samples to identify those with noise that is likely to lead to a click. These samples are erased (i.e. set to zero). The erased points are then interpolated from the remaining good points. The goal, following concepts described in (Taub & Schilling, 1986), is to convert noise *spikes* into lower amplitude noise events without the same impact. These smaller noise events would either avoid significant phase rotation or appear as *doublets*, which do have a phase rotation around the transmitted signal vector, but one that does not encircle the origin.

The interpolated points are then passed on to a standard FM demodulator. The result is a reduction in click noise events and an improvement in demodulated SNR.

3.2. Erasures and Interpolation

The processing is in blocks of N samples. Create a vector s with N input samples from the receiver input $x_{bb}[kT]$ where $T = \frac{1}{f_s}$ is the sampling interval. The input samples are assumed to have an average power of 1 (i.e. $\frac{1}{M}\sum_{k=0}^{M-1}|x_{bb}[kT]|^2 = 1$) which can be implemented with a slow automatic gain control (AGC). Then, starting with the input samples in a column vector:

$$s = \begin{bmatrix} x_{bb}[0] \\ x_{bb}[1] \\ \vdots \\ x_{bb}[N-1] \end{bmatrix} \tag{8}$$

the algorithm then incorporates limiting, targeted erasures, and interpolation of the erased points via the discrete Papoulis-Gerchberg algorithm, following (Ferreira, 1994).

The discrete Papoulis-Gerchberg algorithm is an iterative algorithm that can be used to recover missing samples from a finite length record of band limited data. The version used in this work starts with a band limiting matrix:

$$B = W^H \Gamma W \tag{9}$$

in which W is the NxN discrete Fourier Transform (DFT) matrix, i.e.

$$W = \left(w_n^{ij}\right)_{i,j} \tag{10}$$

from elements $w_n = e^{\frac{2\pi j}{n}}$. and Γ is a diagonal matrix that determines the band of frequencies that are used for interpolation. Since the interpolation is being performed before demodulation, the passed bandwidth is set with values of $\Gamma_n = 1$ for frequencies within the bandwidth determined by Carson's rule (4) and other values of Γ are set to zero.

Multiple iterations of band-limited interpolation are performed over the sample block. The result is smooth values in the samples that had been erased.

The complete algorithm for pre-processing before FM demodulation is summarized below in **Algorithm 1**.

3.3. Simulations and Experiments

To explore the performance benefits of the threshold extension, a simulation examples has been developed and

Algorithm 1 Pre-Demodulation Threshold Extension

Apply limiting to s to generate s_{limit} by:
    $s_{limit}[k_l] = \frac{s[k_l]}{|s[k_l]|}\ \forall\ k_l \mid |s[k_l]| > 1$ and
    $s_{limit}[k_o] = s[k_o]\ \forall\ k_o \mid |s[k_o]| \leq 1$
Create selector matrix $D = diag(d[0]\ d[1]\ \ldots\ d[N-1])$:
    $d[k_e] = 0\ \forall k_e \mid |s_{limit}[k_e]| < \delta$ and
    $d[k_n] = 0\ \forall k_n \mid |s_{limit}[k_n]| \geq \delta$
$x_P = D s_{limit}$
for a = 1 to N_{iters} **do**
    $z_P = B x_P$
    $x_P = D s_{limit} + (I - D) z_P$
end for

tested in GNU Radio. The simulation uses a sine wave baseband signal of $f_m = 1$ KHz and a peak deviation of $k_f = 5$ KHz. The threshold extension processing is implemented in the *papGerch* block (part of the gr-newFM OOT module) with a blocksize $N = 256$ and nulling threshold of $\delta = 0.35$. This is incorporated into the flowgraph in fmDemod_Demo.grc, which plots the demodulated signal power spectral density both with and without the threshold extension. The complete example is publicly available in (Hansen, 2026).

After running the example and giving it time to average, the received noise level plot like the one in Figure 3 is displayed. The plot shows the demodulated signal power spectral density without threshold extension in blue and with threshold extension in red. In the neighborhood of the desired signal, the improvement in SNR with threshold is several dB. In particular, with threshold extension, low frequency noise from clicks are greatly reduced. Also, the threshold extension does not introduce non-linear distortion into the demodulated signal.

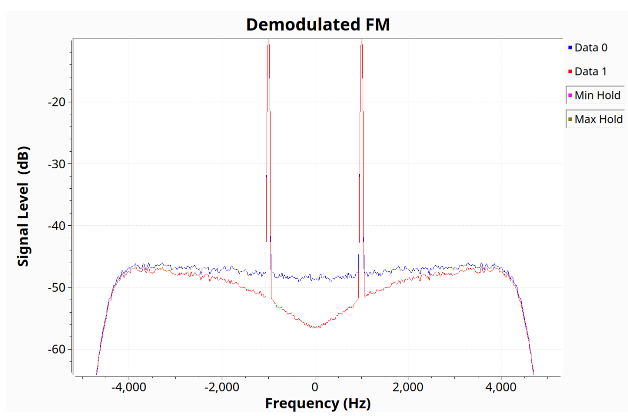

Figure 3. Example of Threshold Extension Improvement

4. Future Work

This work suggests several possible directions for future exploration of threshold extension. While **Algorithm 1** does improve SNR in the demodulated FM signal, the algorithm parameters - block size and nulling threshold level - have not been optimized. Adjustment of these parameters based on input CNR level could offer significant improvements in click reduction.

Another level of performance is likely possibly by improving the method for identifying received sequences that are likely to cause clicks. For example, it may be possible to use feedback from the output of the FM demodulator to chose input samples that are good candidates for erasure. Thus, the threshold extension algorithm becomes iterative, much like modern decoders for error correcting codes.

Ultimately, the author is optimistic that this work will lead to a refined, computationally efficient, open source threshold extended FM demodulator that can be used in many applications.

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