Method for Automatic Calibration of I/Q Amplitude Balance and Quadrature Skew Using GNU Radio

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ABSTRACT

Software Defined Radio (SDR) transmitters that employ digital phase modulation can have imperfections in the transmitted waveform that either distort the intended signal or introduce undesired spurious interference. In the case of zero-intermediate frequency transmission, these impairments include amplitude imbalance between the in-phase and quadrature components, quadrature skew in the vector modulator, and local oscillator feedthrough. In the case of heterodyne transmission, where the digital modulation is applied to a carrier offset before digital-to-analog conversion, an undesired image may be present at the vector modulator output. These transmitter impairments can be observed and measured directly with a vector signal analyzer (VSA), which can also allow the engineer to visualize the digital modulation in the form of constellation plots or eye diagrams. However, if VSA equipment is not available, a series of simple periodic test signals synthesized in the SDR transmitter can allow an engineer to measure the same imperfections directly with a simple spectrum analyzer. This technique can therefore be applied in GNU Radio flowgraphs. In addition, the test signal sequence can also be used to calibrate out the effects with the introduction of predistortion in the transmitter. This further implies that a calibration process can be automated. This paper illustrates the use of a GNU Radio flowgraph in measuring and cancelling out phase modulation imperfections. Emphasis is placed on binary phase shift keying (BPSK), quadrature phase shift keying (QPSK), and offset quadrature phase shift keying (OQPSK) signals. The appropriate test signals to use for each case are identified, and the theoretical spectrum for each signal is derived. Examples of imperfections and the effects they have on the test signal spectrum are shown and interpreted, and the spectral analysis technique measurements are then compared to VSA results.

LOCAL OSCILLATOR FEEDTHROUGH MEASUREMENT BY TRADITIONAL MEANS

A simple modulated quadrature phase shift keyed (QPSK) signal created using a zero-intermediate frequency architecture as shown in Figure 1.

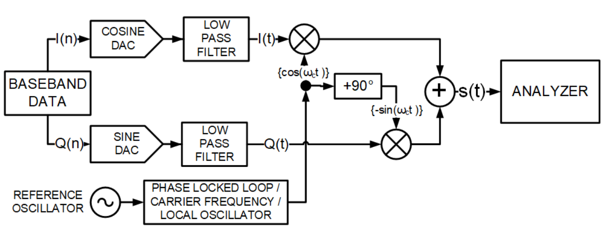

Figure 1: Block Diagram of Vector Modulator Used for Zero Intermediate Fequency Modulation

Two digital-to-analog converters (DACs) create the in-phase I(t) and quadrature Q(t) components of a baseband phase-modulated digital signal. The carrier frequency or local oscillator is generated with a phase-locked loop (PLL), and it synthesizes the carrier at radian frequency ω_c. The PLL output and baseband signal components are passed to a vector modulator, which includes mixers, a phase shifter for the PLL, and a summing junction. The general ideal output is represented in equation (**1**).

$$s(t) = I(t)\cos(\omega_c t) - Q(t)\sin(\omega_c t) \tag{1}$$

There are variations in the implementations of phase shifters in vector modulators. If the phase shifter of Figure 1 were to have a 90-degree phase delay instead of a 90-degree phase advance, the sign of the $Q(t)\sin(\omega_c t)$ term of (**1**) would be positive instead of negative. What matters is that the vector modulator contains a means of keeping the I(t) and Q(t) signals 90 degrees out of phase with respect to one another so that they can be modulated onto the same carrier.

There are different ways that the spectrum of the vector modulator could be measured, depending on the instrument used. A spectrum analyzer typically performs a short duration Fourier transform of the signal after internally down-converting it to baseband. One such instrument spectrum measurement is shown in Figure 2. The figure shows two traces. The blue trace shows the maximum measured power in the resolution bandwidth at each point, and the red trace shows the average power.

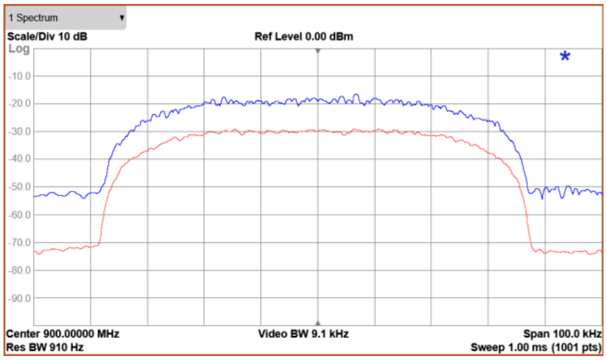

Figure 2: Spectrum of Vector Modulator Output for QPSK Signal 50 kHz Modulation Rate (Blue = MAX HOLD, Red = Average)

If instead, the output of the vector modulator s(t) was connected to a digitizer (with RF down-converter), then the individual digital samples can be processed in MATLAB, and its spectrum can be plotted using the Fast Fourier Transform (FFT) function fft(). The function fftshift() is used to center the spectrum at 0 Hz. This is shown in Figure 3, which is a spectrum of the same signal seen in Figure 2.

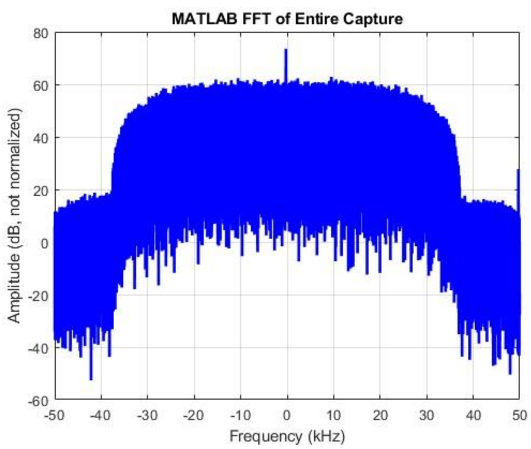

Figure 3: MATLAB Plot of Spectrum of Vector Modulator Output, FFT of Entire Capture (20 Seconds Duration)

Figure 2 and Figure 3 show a similar overall shape, except for a large spike in the center of the spectrum present in Figure 3 but absent in Figure 2. One may suspect that the digitizer that produced Figure 3 has a hardware flaw causing a spectral artifact. There is indeed a hardware flaw, but the flaw is not in the digitizer. The flaw is in the device under test (DUT) itself. There is an imperfection inside the vector modulator that allows part of the local oscillator to appear unmodulated at the output. This is called local oscillator (LO) feedthrough, and it exists in all practical vector modulators ([1], [2]). The MATLAB script file integrates across the entire duration of the transmission. As the integration time increases, the signal's random portion of the magnitude spectrum stops increasing and levels off. However, the non-random carrier feedthrough portion coherently adds with one or two of the FFT bins and thus continues to increase. The magnitude spectrum of the random portion of the signal stops growing with increasing integration time, but the nonrandom carrier feedthrough coherently adds with one or two of the FFT bins, at least for a portion of the integration time. This component of the magnitude spectrum grows with increasing integration time until the phase noise process inherent in the reference oscillator causes the LO feedthrough to change FFT bins. The resulting smearing across a few FFT bins appears as a large spike at the carrier frequency. This can be seen in the close-up view of the example magnitude spectrum, shown in Figure 4. This plot shows that the LO feedthrough is offset from the intended carrier frequency by approximately -281 Hz, and that the carrier drifted between -282 Hz to -280 Hz during the 20-second transmission.

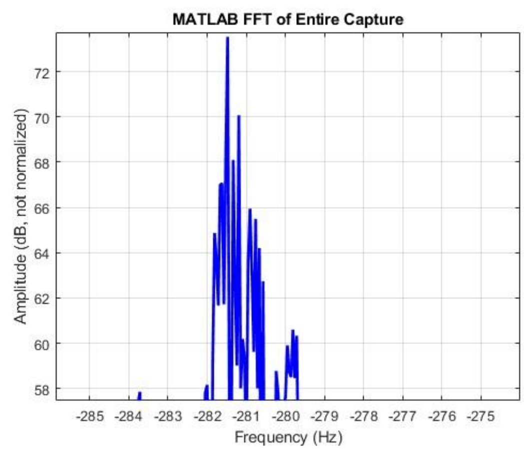

Figure 4: Close-up View of Center Spike Seen in Spectrum Plot of Figure 3, Showing Concentration Between -282 Hz and -280 Hz Offset

The carrier suppression power ratio can be calculated from the sum of squares of the MATLAB spectrum's FFT (see equation (**2**)). The numerator represents the power in the carrier, based on the smeared FFT bins. The denominator represents the signal's total power, in accordance with Parseval's theorem.

$$\begin{matrix} carrier \\ suppression \end{matrix} = \frac{\sum_{\substack{k=peak \\ bins}} |X_{carrier}(k)|^2}{\sum_{k=0}^{k=N-1} |X(k)|^2} \qquad (2)$$

Applying equation (**2**) to this example, the ratio of carrier power to total power is 1.03e8 / 3.73e10 = 2.76e-3. Expressed in decibels, this ratio is -25.59 dB.

A vector signal analyzer can measure carrier suppression without having to perform a long-duration Fourier transform and calculate power ratios. The vector signal analyzer deduces this by computing the center of the constellation plot. The presence of an unmodulated carrier in the form of LO feedthrough causes the constellation to be centered at nonzero in-phase and quadrature coordinates [1]. This measurement is called IQ Offset on the VSA display screen. Figure 5 shows a VSA measurement of -25.035 dB for IQ Offset, which is close to the calculation of -25.59 dB for carrier suppression in the previous discussion. The VSA also shows a frequency error of -280.92 Hz, which agrees with the MATLAB spectrum plot as well.

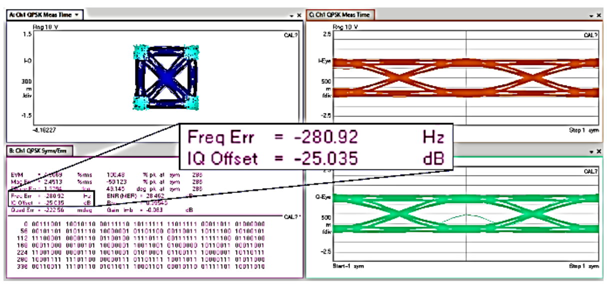

Figure 5: VSA Measurement of Frequency Offset and Carrier Suppression (IQ Offset)

Two different appearances of the QPSK signal's spectrum occurred because of the difference in the duration of the Fourier transform. The short duration typical of most spectrum analyzers is too short to show the presence of the LO feedthrough. The long duration allows a measurement of carrier suppression that agrees with a vector signal analyzer's measurement of the same quantity. Using this measurement technique to calibrate this quantity out of the system requires increasingly longer durations, to reveal the remaining LO feedthrough. The duration of the transform must be sufficient to show the spike at the carrier frequency in the middle of the spectrum. It is more practical to construct a calibration algorithm that employs VSA measurements and applies DC offsets to the transmitter DACs in response. The VSA technique requires locking to the center frequency, locking in time to extract the constellation points, computing the centroids of the constellation points, and calculating the center of the extracted constellation.

A measurement technique that allows a simpler procedure for measuring LO feedthrough (and a quick measurement of frequency offset) is desired. The quantities can be measured using substitute test signals rather taking the measurements directly from the final version of the signal of interest. This paper describes appropriate test signals that allow measurement of carrier suppression, amplitude imbalance, and quadrature skew using only a spectrum analyzer. Measurements using the test signals are compared to the results obtained from direct measurement of modulated signals using a vector signal analyzer.

MEASUREMENT OF CARRIER SUPPRESSION USING TEST SIGNAL

The QPSK signal depicted in Figure 2 through Figure 5 had its LO feedthrough and frequency offset measured in two different ways, one with a vector signal analyzer and one with an FFT of a 20-second digital capture. A third technique is presented in this section which only requires the use of a spectrum analyzer. This technique is to replace the random QPSK signal with a binary phase shift keyed (BPSK) signal consisting of alternating logic 1 and logic 0 bits. Logic 0 maps to -1, and logic 1 maps to +1, so that the signal has no DC offset. For a QPSK signal, this is equivalent to setting I equal to Q and having them both alternate between +1 and -1. The Fourier transform of the signal is a line spectrum, since the signal is periodic. As such, the coefficients of the test signal can be calculated. Let x(t) be the test signal as described, with a period of T seconds. Furthermore, let x(t) be an unfiltered square wave signal. The complex coefficients c_n of its Fourier series are computed by integrating across a period T, as shown in equation (**3**), derived in [3]. The integration is split to reflect the fact that the signal is +1 for the first half period and -1 for the second half period. The transition from +1 to -1 is chosen to occur at time zero, but a different choice for the transition time would only change the phase of the coefficient c_n, not the magnitude.

$$c_n = \frac{1}{T}\left\{ \int_{-\frac{T}{2}}^{0} e^{\frac{-j2\pi nt}{T}}\,dt - \int_{0}^{\frac{T}{2}} e^{\frac{-j2\pi nt}{T}}\,dt \right\} = \frac{2((-1)^n - 1)}{j2\pi n} = \begin{cases} 0, & n\ even \\ \frac{j2}{\pi n}, & n\ odd \end{cases} \tag{3}$$

The test signal contains only odd harmonics of the fundamental frequency 1/T. The ideal line spectrum would be centered at the carrier frequency and consist of lines at ±1/T, ±3/T, ±5/T, and so on. With the signal energy concentrated away from the carrier, anything that remains at the carrier frequency must be the LO feedthrough. The difference in measured power between the unmodulated carrier and the carrier after it is modulated with the test signal is the carrier suppression.

If the spectrum analyzer is equipped with a marker-delta feature, the user can directly measure the carrier suppression as seen in Figure 6, where the blue trace is the unmodulated carrier, and the red trace is the carrier modulated with the alternating logic pattern. A reference marker was placed on the unmodulated peak, and a marker delta measurement was applied to the modulated carrier. The marker delta measurement in the figure reads -26.326 dB, indicating the drop in the carrier's power compared to its unmodulated power level.

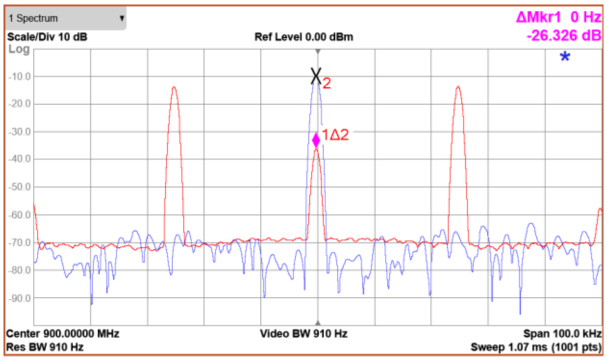

Figure 6: Direct Measurement of Carrier Suppression Using Test Pattern (Red) after Measuring Carrier Power (Blue)

If the spectrum analyzer is not equipped with a marker delta, the carrier suppression can still be approximated from a measurement between the unmodulated carrier and either of the largest adjacent tones plus 3 dB. According to equation (**3**), most of the power is contained in the tones at +1/T or -1/T, so each tone has about half of the power of the modulated signal.

MEASUREMENT OF AMPLITUDE IMBALANCE

Amplitude Imbalance in QPSK

Amplitude imbalance causes a phase modulated constellation to take an oblong shape. For QPSK, it changes from a square to a rectangle. In this paper, amplitude imbalance is represented as a factor α multiplied by the signal emerging from the SINE digital-to-analog converter (DAC) in the vector modulation block diagram. This imperfection is represented in the block diagram shown in Figure 7.

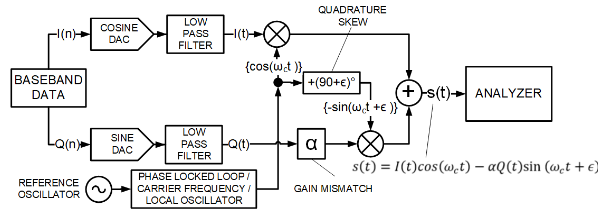

Figure 7: Block Diagram of Vector Modulator with Imperfections in the Analog Circuitry

The location of the gain mismatch may be in the low pass filter or in a branch of the vector modulator itself. The I and Q paths have different gains regardless of the mismatch location.

Direct measurements of amplitude imbalance require separating the I and Q components in frequency so that their individual contributions can be compared on the spectrum analyzer. If one were to simply assign both I and Q to have the alternating [+1 -1 +1 -1…] test pattern, the resulting spectrum would have equal amplitude tones on either side of the suppressed carrier. This is equivalent to simply diagonally moving between two points on the constellation plot. The I and Q components are both present in this signal and cannot be separated. However, if the alternating [+1 -1 …] test pattern is kept on one of the channels and a test pattern changing at half the rate is placed on the other, such as [+1 +1 -1 -1…], then the tones generated will be at different frequency locations. Moreover, the second harmonic of the slower signal will be at the fundamental frequency of the faster signal. Recall from equation (**3**) that the even harmonics of the square wave are zero, so the faster signal's fundamental will appear by itself. With the I and Q components separated from one another, all that remains is to perform power measurements with the spectrum analyzer. For added confidence, the user can alternate between which signal, I or Q, is the faster one and which is the slower one. This gives four measurements of the amplitude imbalance, one for each of the positive and negative fast and slow fundamental tones.

Demonstration of Measurement Technique with USRP

An Ettus Research N210 universal serial radio peripheral (USRP) device was used to simulate a QPSK transmitter that has a 1 dB amplitude imbalance. This is achieved with a simple GNU Radio Companion block diagram, consisting of a constellation object, a USRP sink, and various data generation sources. Figure 8 shows a reproduction of the flowgraph with large print for clarity. The details of the Constellation Rectangular Object definitions for constellation points are listed in subsequent tables.

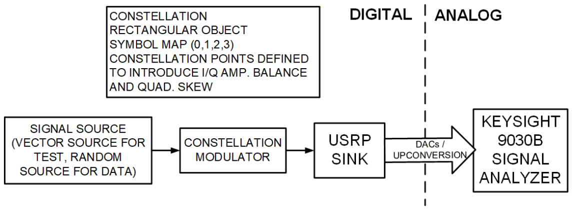

Figure 8: GNU Radio Companion Flowgraph for Testing Amplitude Imbalance Measurement Technique

For simplicity, the constellation points and symbols are defined with I on the real axis, Q on the imaginary axis, and the ordered pairs are defined as (I, Q). Figure 9 shows the constellation point definitions used throughout the paper. It also shows that the constellation points are Gray encoded, with adjacent constellation points differing by only one bit.

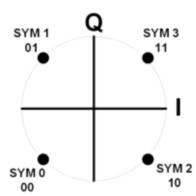

Figure 9: Constellation Point Definitions

The test signals can be produced with a vector source set to output bytes in repeat mode. Constructing the desired test signal requires knowledge of the byte unpacking in the GNU Radio Companion environment. A byte pattern is interpreted as four QPSK symbols with alternating I and Q, with I occupying the most significant bit. Table 1 derives and identifies the repeating byte patterns used measuring amplitude imbalance in GNU Radio. It starts with the individual values for I and Q represented separately in rows and then combines them into two-bit IQ symbols. The IQ symbols of the desired test pattern repeat after four symbols, which conveniently allows a single byte to represent the test pattern. The table translates the four IQ symbols in the repeating pattern as 8-bit decimal values.

Table 1: Generation of Repeating Test Signals in GNU Radio Companion for Measuring Amplitude Imbalance in QPSK Signals

Test Signal	Bit Breakdown		Decimal Representation
I-Fast Q-Slow	I	1 0 1 0	216
	Q	1 1 0 0	
	IQ	11, 01, 10, 00	
I-Slow Q-Fast	I	1 1 0 0	228
	Q	1 0 1 0	
	IQ	11, 10, 01, 00	

The bytes from the vector source are passed to the QPSK modulator. In the GNU Radio environment, the constellation for the modulator was chosen to exhibit 1 dB greater amplitude in the Q-channel than in the I-channel (see Table 2).

Table 2: QPSK Constellation Coordinates for 1-dB Amplitude Imbalance in GNU Radio Constellation Rectangular Object

Symbol Map	Real	Imaginary
0	-0.70711	-0.79339
1	-0.70711	0.79339
2	0.70711	-0.79339
3	0.70711	0.79339

To verify that the constellation has the desired 1-dB amplitude imbalance, a Keysight N9030B PSA Signal Analyzer with Vector Signal Analysis option measured the modulation parameters of the USRP when its data source was set to random. Figure 10 shows that the USRP exhibited an amplitude imbalance of -0.878 dB. The magnitude of the amplitude imbalance is short of the desired 1 dB, but the constellation was only used as a starting point for testing the measurement technique. The USRP has its own inherent amplitude imbalance, and this can contribute to the VSA's measurement. The discrepancy of the sign of the amplitude imbalance can be explained by the fact that the VSA did not know the location of the first constellation point of the burst, and the authors did not add any synchronization symbol or preamble to the signal.

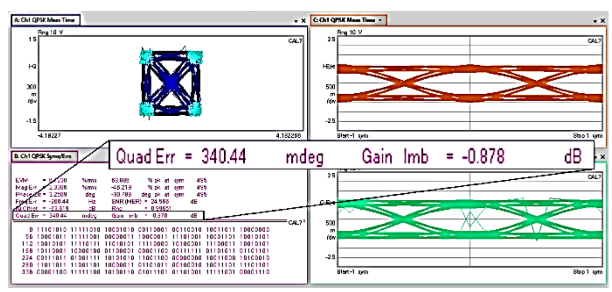

Figure 10: Vector Signal Analyzer Measurements of URSP N210 Using Constellation from Table 2

Next, the data source was changed from random bytes to one of the repeating byte patterns listed in Table 1. The spectrum analyzer was set to center on the carrier frequency, and the span was set to be wide enough to cover the I-Fast and Q-Slow signals simultaneously. The graticule settings for the amplitude on the spectrum analyzer were set to 1 dB per division. Once the USRP began transmitting the test pattern, the spectrum analyzer's trace averaging feature was turned on. The spectrum shows the average of 100 traces of the pattern at four different frequencies, and the power levels measured at those locations. The I-components are twice as far from the center frequency as the Q-components, which can be seen in Figure 11. The markers in the table are arranged in increasing frequency order.

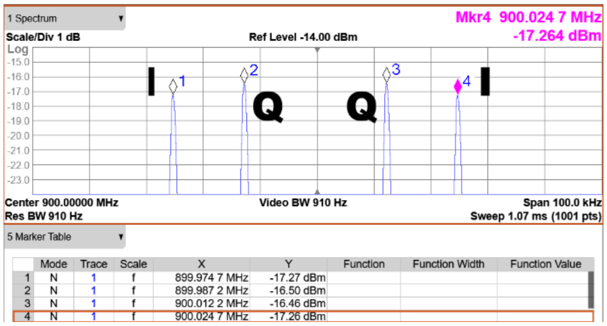

Figure 11: Spectrum and Tone Power Measurement for I-Fast Q-Slow Test Pattern

The test pattern was changed to the I-Slow Q-Fast pattern, and the spectrum was measured again. Once again, the spectrum trace averaging feature was turned on with a factor of 100. The resulting spectrum is shown in Figure 12, including a marker table.

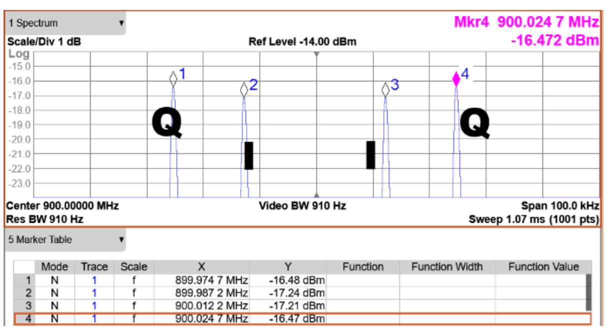

Figure 12: Spectrum and Tone Power Measurement for I-Slow Q-Fast Test Pattern

The differences between the I-component and Q-component power levels are tabulated in Table 3. The measurements range between 0.74 dB and 0.79 dB, which are within 0.14 dB of the VSA measurement shown in Figure 10. The spectrum analyzer measurement technique has an advantage over the VSA measurement where it can identify whether I or Q is larger without having to input a synchronization pattern.

Table 3: Amplitude Imbalance Measurements from Figure 11 and Figure 12

Freq.	I Power (dBm)	Q power (dBm)	Q-I Difference (dB)
899.9750 MHz	-17.27	-16.48	0.79
899.9875 MHz	-17.24	-16.50	0.74
900.0125 MHz	-17.21	-16.46	0.75
900.0250 MHz	-17.26	-16.47	0.79

The measurement technique is intended to be independent of quadrature skew inherent in the vector modulator. Quadrature skew can be described as any deviation in the vector modulator's local oscillator phase shifter from a magnitude of 90 degrees ([1], [2], [4]). A deviation from 90 degrees causes crosstalk between the I(t) and Q(t) components of the signal ([1],[2],[4]). The deviation causes the phase modulated constellation to appear as a rhombus (or a parallelogram, if there is also amplitude imbalance). In addition, the frequency domain components of I(f) and Q(f) have a rotational angle between them that deviates from ideal quadrature, as seen in the example shown in Figure 13. However, because the peaks of I(f) align with the nulls of Q(f) and vice versa, there is no "crosstalk" in the frequency domain. The figure illustrates that the frequency components of I and Q can be measured independently regardless of the angle between them because they are already separated in frequency.

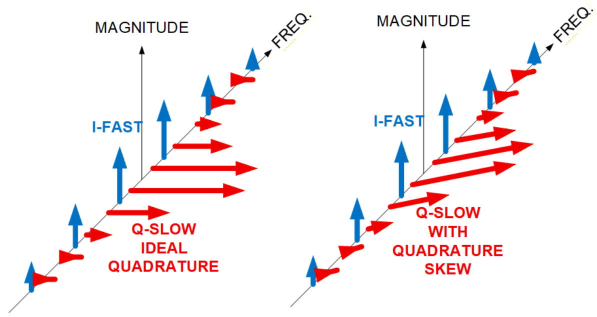

Figure 13: Effect of Quadrature Skew on I-Fast, Q-Slow Test Signal as Seen in Frequency Domain

Amplitude Imbalance in Offset QPSK (OQPSK)

Offset QPSK signals prohibit simultaneous change on the I and Q channels. The periodic signals devised in the previous section possess simultaneous I and Q changes, causing a trajectory through the origin of the constellation. Therefore, the periodic signals from the previous section must be modified to measure amplitude imbalance on OQPSK signals. The simplest fix is to set one of the channels, I or Q, to be constant while the other one varies in an alternating logic level. Because one of the channels is constant, its output is located at the center frequency of the carrier. The changing channel has tones located on either side of the carrier. Apart from this key difference, the technique for measuring amplitude imbalance in OQPSK signals is the same as the procedure for QPSK signals. Table 4 shows modified repeating byte patterns for GNU radio to be used for OQPSK signals.

Table 4: Generation of Repeating Test Signals in GNU Radio Companion for Measuring Amplitude Imbalance in OQPSK Signals

Test Signal	Bit Breakdown		Decimal Representation
I-Fast Q-Slow (Q constantly high)	I	1 0 1 0	221
	Q	1 1 1 1	
	IQ	11, 01, 11, 01	
I-Slow Q-Fast (I constantly high)	I	1 1 1 1	238
	Q	1 0 1 0	
	IQ	11, 10, 11, 10	

To demonstrate the measurement technique on a deliberately impaired OQPSK signal, the GNU Radio Companion block diagram test setup for the USRP N210 was changed from its QPSK block diagram. A means of introducing the necessary half symbol time delay between the in-phase and quadrature channels needed to be implemented. Rather than introduce the errors in the constellation itself, the constellation was assigned to be ideal, and the errors were introduced later in the chain after the half-symbol time delay is created. This is necessary because once quadrature skew is introduced, it is no longer possible to separate the I and Q components for the purpose of delaying the Q-component. Figure 14 shows that crosstalk is introduced by the factors ϵ_I and ϵ_Q, and amplitude imbalance is introduced from factors α_I and α_Q. To create a magnitude of ϵ radians of quadrature skew, the values of ϵ_I and ϵ_Q could be assigned to ϵ and 0 (or 0 and ϵ), respectively. An alternative is to assign both ϵ_I and ϵ_Q to $\epsilon/2$.

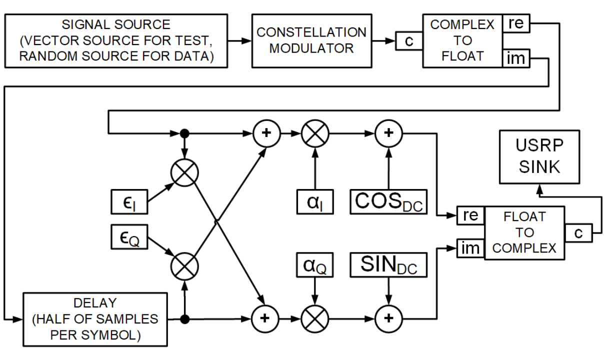

Figure 14: Block Diagram for Generating OQPSK Signals with Amplitude Imbalance and Quadrature Skew Distortion

The DC controls COS_{DC} and SIN_{DC} are for controlling LO feedthrough. They add DC offsets right before the COSINE DAC and SINE DAC, respectively. This changes the location of the constellation origin.

Table 5 lists the parameters settings used in the GNU Radio Companion block diagram to generate a 1-dB amplitude imbalance and a 2-degree quadrature skew. The parameters ϵ_I and ϵ_Q are assigned to be equal, each contributing one degree of quadrature skew. The ratio of the amplitudes α_Q / α_I provides the 1-dB of amplitude imbalance. These settings were applied to a random signal, and its constellation pattern and measurements were taken with the VSA, as show in Figure 15.

Table 5: OQPSK Settings for 1-dB Amplitude Imbalance and 2-Degree Quadrature Skew in GNU Radio Using Structure of Figure 14

Distortion Parameter	Value
ϵ_I	0.017453
ϵ_Q	0.017453
α_I	0.500
α_Q	0.561
COS_{DC}	0
SIN_{DC}	0

The VSA measurements shown in Figure 15 are in close agreement with the desired settings of 2 degrees quadrature skew and 1-dB amplitude imbalance.

Next, the USRP data source was switched from random data to the I-Fast Q-Slow signal from Table 4, followed by the I-Slow Q-Fast signal from the same table. The spectra and measurements are in Figure 16 and Figure 17. The amplitude imbalance measurements taken with the different test signals are shown in Table 6.

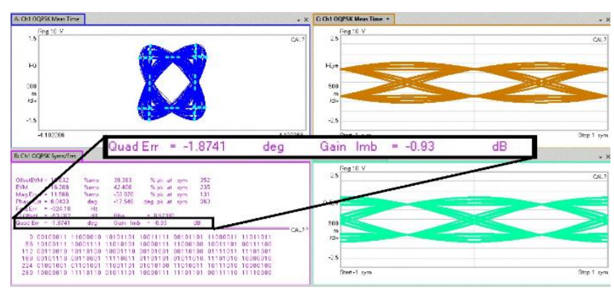

Figure 15: VSA Measurement of USRP with OQPSK Signal Using Parameters in Table 5

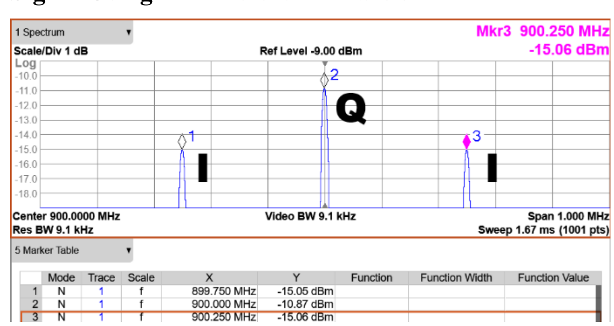

Figure 16: I-Fast Q-Slow OQPSK Test Spectrum and Measurements

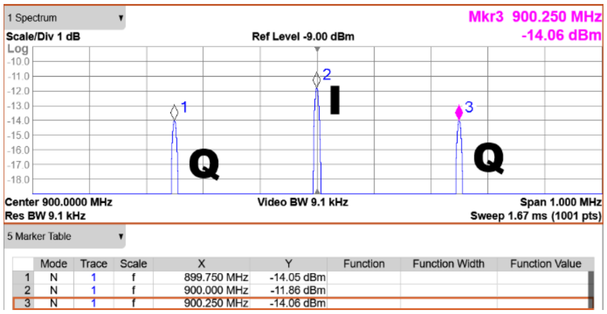

Figure 17: I-Slow Q-Fast OQPSK Test Spectrum and Measurements

Table 6: Amplitude Imbalance Measurements from Figure 16 and Figure 17

Freq.	I Power (dBm)	Q power (dBm)	Q-I Difference (dB)
899.9750 MHz	-15.05	-14.05	1.00
900.0000 MHz	-11.86	-10.87	0.99
900.0250 MHz	-15.06	-14.06	1.00

The measurements for amplitude imbalance using this technique were very close to the VSA measurement, within hundredths of a decibel. The test signals from Table 4 were specifically designed for OQPSK modulators, but they can also be used for ordinary QPSK modulators. Alternatively, a QPSK modulator can be checked with signals from both Table 1 and Table 4 for added confidence of measured values.

MEASUREMENT OF QUADRATURE SKEW

A convenient periodic signal for measuring quadrature skew is a complex sinusoid at a frequency offset from the center of the band. This complex sinusoid will pass through the vector modulator, and the presence of quadrature skew will be revealed when an image appears at the negative of the chosen frequency offset. It is permissible for the test signal to have harmonics provided none are located at the negative of the fundamental. The simplest QPSK test signal that fulfills these requirements is constructed by increasing (or decreasing) the phase by 90 degrees at every symbol period. Starting at symbol 0 in Figure 9, the generation of a positive fundamental frequency offset requires counterclockwise traversal of the constellation, using the symbol sequence (0, 2, 3, 1). Traversing the constellation in the clockwise direction (0, 1, 3, 2) generates a negative fundamental frequency offset. Since these symbols are shaped pulses rather than sinewaves, the output of the vector modulator will have odd harmonics of the cycle rate. In addition, the signs of the odd harmonics alternate. There are four symbols per cycle, and the GNU Radio Constellation Modulator is set to four samples per symbol. This product is 16 samples per cycle. Therefore, the fundamentals will be located at 1/16 of the sample rate with a positive or negative sign depending on the direction of phase traversal. Table 7 derives the bytes to use in the vector source before the constellation modulator block for both positive and negative fundamental frequency generation.

Table 7: Generation of Repeating Test Signals in GNU Radio Companion for Measuring Quadrature Skew in QPSK Signals

Test Signal	Bit Breakdown		Decimal Representation
Positive Fundamental Frequency Offset	I	0 1 1 0	45
	Q	0 0 1 1	
	IQ	00, 10, 11, 01	
Negative Fundamental Frequency Offset	I	0 0 1 1	30
	Q	0 1 1 0	
	IQ	00, 01, 11, 10	

Figure 18 shows the spectrum and constellation plots that occur when the vector source to the constellation modulator is repeating number 45, which produces the positive frequency offset. In this example, the sample rate is 32 kHz, and there are tones at 2 kHz (1/16 of the sample rate), -6 kHz (negative 3rd harmonic), 10 kHz (positive 5th harmonic), and -14 kHz (negative 7th harmonic). For measurements, the fundamental at 2 kHz would be used. The absence of a tone at -2 kHz prevents the signal from masking the quadrature skew measurement. In addition, the signal does not produce a DC term that would mask the LO feedthrough.

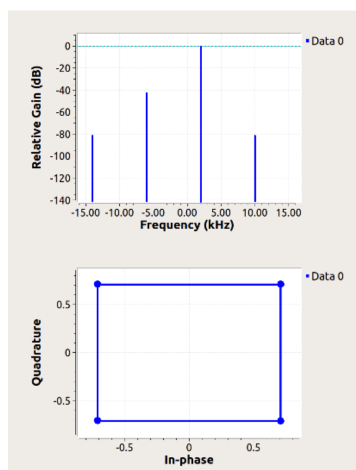

Figure 18: Spectrum and Constellation Resulting from Using Repeating Byte Value 45 in QPSK Modulator

The values in Table 7 will not work for OQPSK signals in GNU Radio. The half-symbol delay in OQPSK causes signals generated from Table 7 to exhibit amplitude modulation where the constellation is an ellipse rather than a circle. The amplitude cycles through its values twice for every cycle of the constellation, which means its frequency content surrounds the desired fundamental frequency with content at twice that frequency. Therefore, a tone appears at the negative of the fundamental, preventing an accurate measurement of quadrature skew.

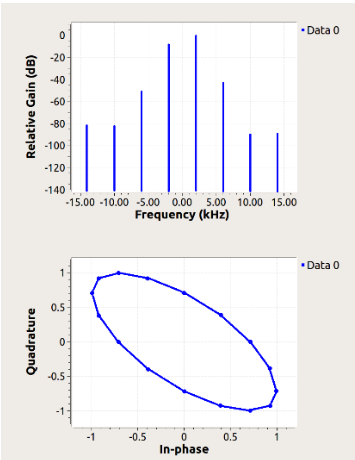

Figure 19: Spectrum and Constellation Resulting from Using Repeating Byte Value 45 in OQPSK Modulator

Instead, the I and Q channels should be assigned to alternate values on every symbol. Then, the half-symbol delay causes the resulting signal to trace out the constellation with a constant radius, as desired. Table 8 lists the proper values to use for OQPSK, and it shows how to make positive and negative fundamental tones. The spectrum and constellation plots for the positive frequency offset signal are shown in Figure 20.

Table 8: Generation of Repeating Test Signals in GNU Radio Companion for Measuring Quadrature Skew in OQPSK Signals

Test Signal	Bit Breakdown		Decimal Representation
Positive Fundamental Frequency Offset	I	1 0 1 0	204
	Q	1 0 1 0	
	IQ	11, 00, 11, 00	
Negative Fundamental Frequency Offset	I	1 0 1 0	153
	Q	0 1 0 1	
	IQ	10, 01, 10, 01	

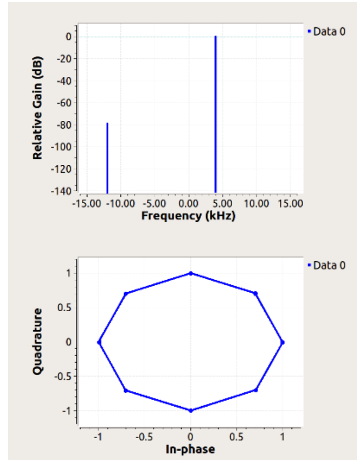

Figure 20: Spectrum and Constellation Resulting from Using Repeating Byte Value 204 in OQPSK Modulator

For both QPSK and OQPSK, the test signals are designed to create a fundamental tone on one side of the carrier frequency. Quadrature skew causes an image to appear at the negative of the fundamental tone. The amplitude of the desired signal relative to the image is related to the quadrature skew and the amplitude imbalance. If the amplitude imbalance is already known from previous measurements, the magnitude of the quadrature skew can be solved. In the Appendix, the case of digital up-conversion is treated. First, it demonstrates that the output contains only the upper sideband in the absence of amplitude imbalance and quadrature skew. Next, the expressions for the upper sideband and lower sideband signals are derived for the case of the presence of imbalance and skew. Finally, the expression for the power ratio between the upper and lower sidebands, also known as the image rejection ratio (IRR), is derived as equation (A.10). The same equation can be rearranged to solve for the square of the quadrature skew, as shown in (**4**).

$$\epsilon^2 = \frac{IRR(1 - 2\alpha + \alpha^2) - (1 + 2\alpha + \alpha^2)}{\alpha^2(1 - IRR)} \qquad (4)$$

Taking the square root of ϵ^2 produces the magnitude of the quadrature skew expressed in radians. The technique does not resolve the sign ambiguity of the quadrature skew.

Demonstration of Quadrature Skew Measurement with USRP

The first step in measuring quadrature skew with a spectrum analyzer is to measure the amplitude imbalance, α. This was done in the previous section for an OQPSK signal using the block diagram in Figure 14 and the settings shown in Table 5. The up-conversion frequency was set to 250 kHz, which is 1/8 of the sample rate of 2MHz. To continue that example using the previously measured amplitude imbalance α = 0.99 dB = 1.1207, the signals were replaced with a vector source repeating a byte value of 204, corresponding to a positive fundamental frequency offset. The resulting spectrum is shown in Figure 21, revealing an IRR of 24.457 dB = 279.062.

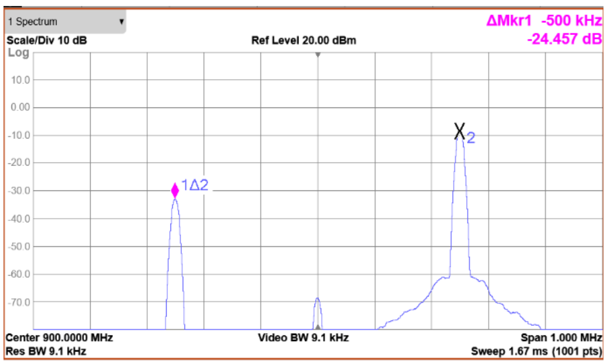

Figure 21: Spectrum of USRP Configured as in Figure 14 Using Settings from Table 5

Applying equation (**4**) produces ϵ = 0.0351 radians = 2.0107 degrees. This magnitude is within 0.15 degrees of the measured value of -1.8741 degrees using the VSA (see Figure 15).

CORRECTION OF AMPLITUDE IMBALANCE AND QUADRATURE SKEW USING DIGITAL UPCONVERSION OR DOWNCONVERSION

A trivial way to remove amplitude imbalance and quadrature skew from a phase modulated signal is to apply digital up-conversion or down-conversion and simply filter out the undesired image. When the baseband phase modulated data is multiplied by a complex sinusoid in the digital domain, both I and Q appear on the COSINE DAC and the SINE DAC. In other words, both I and Q experience the same amplitude and quadrature imperfections. In fact, the effects of quadrature skew and amplitude imbalance are removed from the constellation on both the upper sideband and lower sideband. See the Appendix section titled "Effect of Digital Frequency on Constellation" for detailed derivations for both sidebands.

USING PREDISTORTION TO MANUALLY CORRECT MODULATION IMPERFECTIONS

A common spectrum analyzer can assist with calibration of LO feedthrough, amplitude imbalance, and quadrature skew in a GNU Radio flowgraph. The flowgraph must include in its USRP sink path a set of pre-distortion values that can be dynamically changed, as in the case of using Q-Toolkit (QT GUI) sliders and counters. An example of such a structure is shown in Figure 22. Predistortion deliberately introduces DC offsets, crosstalk, and different gain values for the I and Q channels which cancel out the effects of the analog vector modulator. The spectrum analyzer is monitored as one varies each individual parameter one at a time by manipulating either its associated slider or counter. The quadrature skew is changed until the undesired sideband is minimized. Next, the amplitude imbalance is adjusted to minimize the undesired sideband image further. Finally, the DC offsets are selected until the carrier feedthrough is minimized.

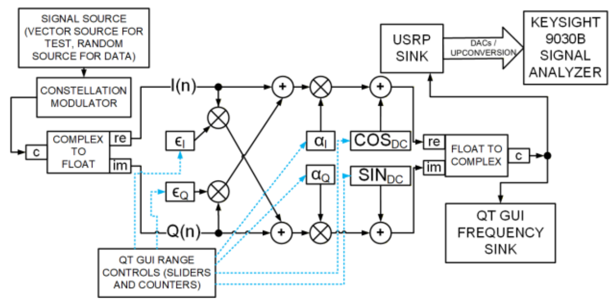

Figure 22: GNU Radio Flowgraph for Manual Calibration

This procedure was demonstrated with the USRP N210 used throughout this paper. This time, however, no deliberate impairments were introduced. The block diagram of Figure 22 was used to generate the test pattern with all the α, ϵ, and DC offset values set to zero. The resulting spectrum of the test pattern is seen in Figure 23. This represents the performance of the USRP N210 without calibration. The figure shows that the LO feedthrough tone is already being suppressed by greater than 60 dB, and the image rejection ratio is measured as 47.9 dB.

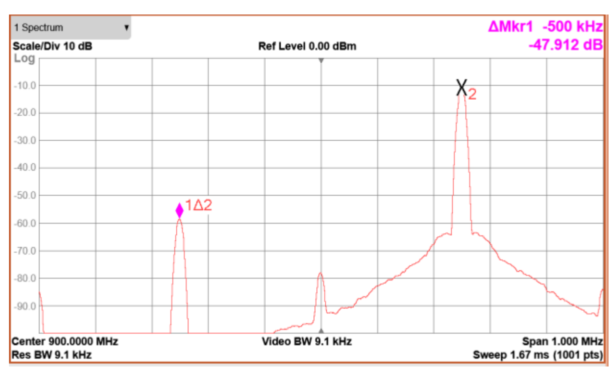

Figure 23: USRP Test Signal Spectrum Before Calibration

Next, the values of ϵ and α were adjusted to minimize the lower sideband tone, and the values of COS_{DC} and SIN_{DC} were adjusted to minimize the LO feedthrough tone in the center of the spectrum. The values shown in Table 9 yielded the best results.

Table 9: Parameter Values that Minimize Undesired Image and LO Feedthrough

Distortion Parameter	Value
ϵ_I	0.0022 (0.126 degrees)
ϵ_Q	0.0000
α_I	0.992200
α_Q	1.000000
COS_{DC}	-0.00040
SIN_{DC}	0.00000

If these tabulated values are applied to the equation for the image rejection ratio (equation (A.12)), the returned value is 47.8123 dB, which is with 0.1 dB of the measured value from the spectrum analyzer. The equation for IRR uses α as the ratio of the Q-amplitude over the I-amplitude, so a value of $\alpha = \alpha_Q/\alpha_I = 1/0.9922 = 1.0079$ must be used.

The spectrum of the up-converted sinusoidal test pattern after the optimum adjustments were found is shown in Figure 24. The LO feedthrough is now below the skirt of the upper sideband, and the LO feedthrough is below the noise level of the spectrum analyzer. The carrier suppression is visibly improved by 15 dB, and the image rejection ratio improved to nearly 90 dB.

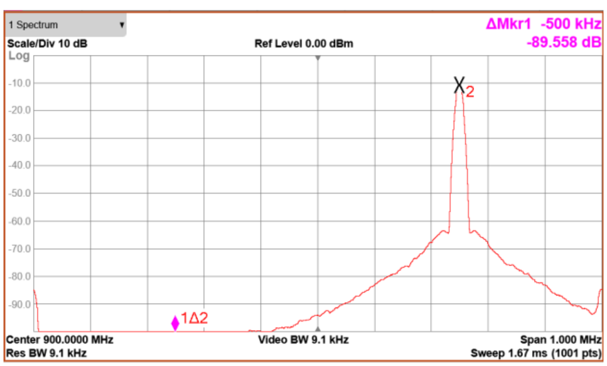

Figure 24: USRP Test Signal Spectrum After Calibration

The test signal was replaced with ordinary baseband random OQPSK data, and the resulting VSA measurement is shown in Figure 25. The quadrature skew is in tens of milli-degrees, and the amplitude imbalance is only 0.025 dB. The measured value of the carrier suppression is greater than 60 dB, but this measurement is affected by the presence of non-zero mean data during the VSA's observation interval. To observe the improvement in the USRP's carrier suppression following calibration, the random data would need to have equal representation of all constellation points during the slice of time that the VSA computed performance statistics.

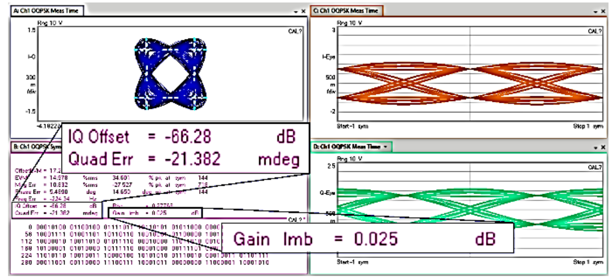

Figure 25: VSA Measurement of USRP After Calibration Using Random Data

GNU RADIO AUTOMATED CALIBRATION OF QPSK TRANSMISSIONS

The previous section involved manually adjusting amplitude imbalance, DC offsets, and quadrature predistortion with a human in the loop, viewing the spectrum on an external spectrum analyzer. This section describes an implementation of a GNU Radio Modulation Calibration Utility built around an Embedded Python Block that processes a signal received on RF 2 port of an N210 USRP. The flowgraph also makes use of an Out of Tree (OOT) module that allows asynchronous control of a stream output from many stream inputs. Other OOT modules provide a means to allow the Embedded Python block to feed correction terms and factors to the calibration structure that removes LO feedthrough, amplitude imbalance, and quadrature skew in the transmit USRP Sink path. Polymorphic type (PMT) messages provide a convenient, asynchronous means of passing information from the Embedded Python Block to other locations in a flowgraph.

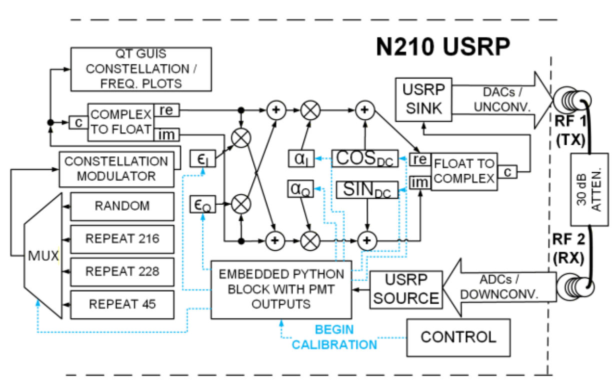

Figure 26: Automated Calibration of GNU Radio USRP N210 Transmitter for QPSK Signals

Figure 26 shows a diagram of an automated calibration flowgraph for QPSK signals. An external 30 dB attenuator is placed between the transmit and receive ports of the USRP to prevent damage to the receive port. Initially, the mux block has the random data source selected, allowing the user to witness the spectrum and constellation plots for the system before calibration is applied. At the same time, the Embedded Python block polls the state of a control from a QT GUI. When a control from a QT GUI is pressed, the Embedded Python block switches to test signals. First, it alternates back and forth between the repeat 216 (I-Fast Q-Slow) signal and the repeat 228 (I-Slow Q-Fast) signal, and then uses the peaks extracted from the spectra of each signal to measure the amplitude imbalance. The *fftlen* parameter of the Embedded Python Block controls the size of the FFT used to extract the peaks. The *sps* parameter is the number of samples per symbol, and is the same value used in the Constellation Modulator block. The spectrum is computed with numpy.fft.fft(), while being centered around the carrier frequency with the numpy.fft.fftshit() function. The process cyclically shifts the spectrum array, placing the negative frequencies at the beginning of the array and positioning the zero-frequency element in the center. The Constellation Modulator Block uses 4 samples per symbol. Thus, the fast tones are located at 1/8 of the sample rate, and the slow tones are located at 1/16 of the sample rate. The frequency vector bin numbers of the peaks are computed from equation (**5**), where $\lfloor\,\rfloor$ is the floor operator that rounds the value down to the nearest integer.

$$freq.index = \left\lfloor frequency * \frac{fft\ len}{f_s} + \frac{fftlen}{2} \right\rfloor \qquad (5)$$

The approximate locations in the frequency array can be deduced by applying equation (**5**) to the fast and slow test signal frequencies. Table 10 shows the locations for *sps*=4 and *fftlen*=8192.

Table 10: Frequency Bin Locations for QPSK Amplitude Imbalance Test Signal Using 8192-Point FFT and 4 Samples per Symbol

Test Signal Frequency Tone	Expected Bin Location	Bin Search Range for Peak
Fast Minus	3072	2816 – 3328
Slow Minus	3584	3328 – 3840
DC	4096	3840 – 4352
Slow Plus	4608	4352 – 4864
Fast Plus	5120	4864 – 5376

The Embedded Python Block applies a restricted search for the principal peaks of the test signal. The search range is the expected location plus or minus half the number of bins between the signification tones. For an FFT of length 8192, there are 512 bins between the different test signal tones. Therefore, the search range is established as the plus or minus 256 bins.

With a calculated value for the imbalance, the Embedded Python Block compares the value to an acceptable minimum threshold. If the measured imbalance exceeds the threshold, the Embedded Python Block adjusts the value on the α_I port (if I is higher in amplitude than Q) or the α_Q port (if Q is higher in amplitude than I) using PMT messages. This scheme allows amplitude corrections to avoid amplifying either path. By restricting the amplitude adjustments to reductions instead of amplifications, the system avoids the possibility of overdriving an input and generating intermodulation distortion products. If the transmitted signal level is low enough that this is not a danger, the code could be adjusted to apply changes to only one path, I or Q.

The Embedded Python Block switches to measuring and cancelling quadrature skew once the amplitude imbalance has been confirmed to be below the accepted threshold. The Python block sends a command to the mux block to change its output to a vector source repeating byte code 45, generating a positive desired tone at 1/16 of the sample rate. It searches for the positive desired tone and the negative undesired image using restricted search bin ranges. For an 8192-point FFT, the ranges are shown in Table 11. The search ranges for the positive and negative tones in Table 11 are the same ranges used for the slow tones in the amplitude imbalance test peak searches of Table 10, because the features to search for are at the same locations.

Table 11: Frequency Bin Locations for QPSK Quadrature Skew Test Signal Using 8192-Point FFT and 4 Samples per Symbol

Test Signal Frequency Tone	Expected Bin Location	Bin Search Range for Peak
Negative Image	3584	3328 – 3840
DC	4096	3840 – 4352
Positive Fundamental	4608	4352 – 4864

The Embedded Python Block extracts the peaks, computes the IRR value, and applies equation (**4**) to find the magnitude of the square of the quadrature skew. Initially, the block temporarily assumes that the quadrature skew is positive. It computes an adjustment halfway between the current computed value and the value applied to the ϵ_Q port of the diagram. Then, the block remeasures the quadrature skew. If the IRR gets worse, the Embedded Python Block changes the sign of the new correction applied to ϵ_Q. The process continues until the IRR has diminished below a threshold.

During the quadrature skew correction routine, the Embedded Python Block applies corrections to the LO feedthrough by locating the DC peak. The process adjusts COS_{DC} and SIN_{DC} to minimize the feedthrough below a threshold. The LO feedthrough adjustment step has been committed to the quadrature skew portion of the calibration because both the QPSK and OQPSK quadrature skew test signals avoid putting a tone on DC.

MODIFICATION OF GNU RADIO AUTOMATED CALIBRATION FLOWGRAPH FOR OQPSK TRANSMISSIONS

The use of OQPSK requires changing the I/Q amplitude balance vector sources to repeating value of 221 and a repeating value of 238. The quadrature skew test signal is changed to a repeating value of 204. The transmit path must also have a delay on the imaginary portion of the generated signal equal to one half symbol, or half the number of samples per symbol. These changes are reflected in the modified diagram shown in Figure 27.

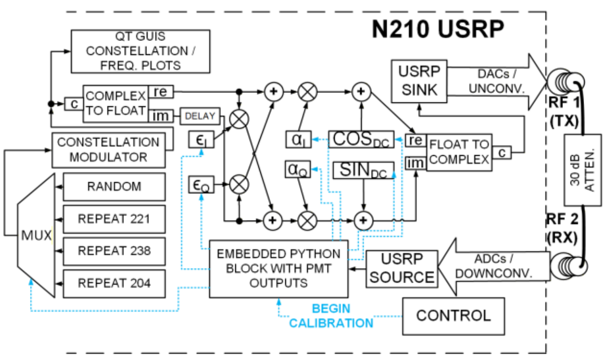

Figure 27: Automated Calibration of GNU Radio USRP N210 Transmitter for OQPSK Signals

Table 12 lists the test signals' FFT search bins for both amplitude imbalance and quadrature skew for OQPSK signals. The table shows that it is not possible to tune LO feedthrough while the I/Q amplitude imbalance portion of the calibration routine is executing because the "slow" amplitude imbalance tone occupies the DC position.

Table 12: Frequency Bin Locations for OQPSK Amplitude Imbalance and Quadrature Skew Test Signals Using 8192-Point FFT and 4 Samples per Symbol

I/Q Amp. Imbalance Signal Part	Quad. Skew Signal Part	Expected Bin Location	Bin Search Range for Peak
Fast Minus	Negative Image	3584	3328 – 3840
Slow	DC	4096	3840 – 4352
Fast Plus	Positive Fundamental	4608	4352 – 4864

CONCLUSION

Imperfections in digital phase modulation can prevent a communication link from achieving its theoretical bit error rate for a given signal to noise ratio. These imperfections ordinarily cannot be seen by inspecting a spectrum analyzer plot, but they can be exposed with the careful selection of periodic test signals. The measurement techniques explained in this paper only require a spectrum analyzer. In addition, various tests performed in the previous sections demonstrate that the measurements compare favorably to the same quantities measured with a vector signal analyzer. The difficulty of measuring constellation points and applying geometric expressions to derive amplitude imbalance and quadrature skew is replaced with the simplicity of measuring narrow peaks in a spectrum to obtain the same results. To streamline the process, a demonstration GNU Radio flowgraph built around an Embedded Python Block was used to demonstrate an automated calibration capability if one stays within the GNU Radio Companion environment.

ACKNOWLEDGEMENTS

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APPENDIX

The vector modulator scheme represented in Figure 1 can be used to simultaneously up-convert (or down-convert) the PLL carrier frequency and modulate the converted frequency. The ability to perform frequency up-conversion or down-conversion is limited by the sample rate inside the baseband signal generator (and the DACs). The two mixers in the vector modulator will generate sum and difference frequencies. With no quadrature skew and no amplitude imbalance, one of the frequencies will coherently add and the other will cancel, leaving a single modulated sideband.

Suppose the up-conversion operation is desired. This means that the I(t) + jQ(t) baseband samples of data must be up-converted by exp(jω_nt).

$$baseband = (I(t) + jQ(t))(e^{j\omega_n t})\Big|_{t=\frac{n}{f_s}} \quad \textbf{(A.1)}$$

The real part of this product is passed to the cosine DAC, and the imaginary part is passed to the sine DAC.

$$COSINE\ DAC = I(t)\cos(\omega_n t) - Q(t)\sin(\omega_n t) \quad \textbf{(A.2)}$$

$$SINE\ DAC = Q(t)\cos(\omega_n t) + I(t)\sin(\omega_n t)$$

The output of the vector modulator is shown in equation (A.3).

$$s(t) = [I(t)\cos(\omega_n t) - Q(t)\sin(\omega_n t)] \cdot \cos(\omega_c t) - [Q(t)\cos(\omega_n t) + I(t)\sin(\omega_n t)] \cdot \sin(\omega_c t) \quad \textbf{(A.3)}$$

Using trigonometric identities to compute the products of sines and cosines allows a decomposition in terms of sum and difference frequencies. The identities used are

$$\mathrm{I(t)}\cos(\omega_n t)\cos(\omega_c t) = \frac{I(t)}{2}\cos\big((\omega_n - \omega_c)t\big) + \frac{I(t)}{2}\cos\big((\omega_n + \omega_c)t\big) \quad \textbf{(A.4)}$$

$$-\mathrm{I(t)}\sin(\omega_n t)\sin(\omega_c t) = \frac{-I(t)}{2}\cos\big((\omega_n - \omega_c)t\big) + \frac{I(t)}{2}\cos\big((\omega_n + \omega_c)t\big)$$

$$-\mathrm{Q(t)}\sin(\omega_n t)\cos(\omega_c t) = \frac{-Q(t)}{2}\sin\big((\omega_n - \omega_c)t\big) + \frac{-Q(t)}{2}\sin\big((\omega_n + \omega_c)t\big)$$

$$-Q(t)\cos(\omega_n t)\sin(\omega_c t) = \frac{Q(t)}{2}\sin\big((\omega_n - \omega_c)t\big) - \frac{Q(t)}{2}\sin\big((\omega_n + \omega_c)t\big)$$

The carrier frequency ω_c is greater than the NCO frequency ω_n, and it is customary to change the (ω_n-ω_c) terms into (ω_c-ω_n) terms with the sine terms changing signs. However, the approach taken here is to keep the order of the frequency difference as (ω_n-ω_c) to minimize sign changes. This simplifies the decomposition and analysis of the expressions.

When the terms of (A.4) are added together, the difference frequency terms cancel out, and the sum frequency terms combine. The result is shown in equation (A.5)

$$s(t) = I(t)\cos\big((\omega_n + \omega_c)t\big) - Q(t)\sin\big((\omega_n + \omega_c)t\big) \quad \textbf{(A.5)}$$

Now consider the presence of an amplitude imbalance factor α in the SINE DAC path, and quadrature skew ϵ in the vector modulator. Equation (A.3) changes to the expression shown in (A.6).

$$s(t) = [I(t)\cos(\omega_n t) - Q(t)\sin(\omega_n t)] \cdot \cos(\omega_c t) - \alpha[Q(t)\cos(\omega_n t) + I(t)\sin(\omega_n t)] \cdot \sin(\omega_c t + \epsilon) \quad \textbf{(A.6)}$$

Once again, the products of signs and cosines are re-expressed as sum and difference frequencies using trigonometric identities. The individual terms are shown in (A.7).

$$\mathrm{I(t)}\cos(\omega_n t)\cos(\omega_c t) = \frac{I(t)}{2}\cos\big((\omega_n - \omega_c)t\big) + \frac{I(t)}{2}\cos\big((\omega_n + \omega_c)t\big) \quad \textbf{(A.7)}$$

$$\begin{aligned}-\alpha\mathrm{I(t)}\sin(\omega_n t)\sin(\omega_c t + \epsilon) &= \frac{-\alpha I(t)}{2}\cos\big((\omega_n - \omega_c)t - \epsilon\big) + \frac{\alpha I(t)}{2}\cos\big((\omega_n + \omega_c)t + \epsilon\big)\\ &\approx \frac{-\alpha I(t)}{2}\cos\big((\omega_n - \omega_c)t\big) - \frac{\alpha\epsilon I(t)}{2}\sin\big((\omega_n - \omega_c)t\big) + \frac{\alpha I(t)}{2}\cos\big((\omega_n + \omega_c)t\big) - \frac{\alpha\epsilon I(t)}{2}\sin((\omega_n + \omega_c)t)\end{aligned}$$

$$-\mathrm{Q}(t)\sin(\omega_n t)\cos(\omega_c t) = \frac{-Q(t)}{2}\sin\big((\omega_n-\omega_c)t\big) + \frac{-Q(t)}{2}\sin\big((\omega_n+\omega_c)t\big)$$

$$\begin{aligned}-\alpha Q(t)\cos(\omega_n t)\sin(\omega_c t+\epsilon) &= \frac{\alpha Q(t)}{2}\sin\big((\omega_n-\omega_c)t-\epsilon\big) - \frac{\alpha Q(t)}{2}\sin\big((\omega_n+\omega_c)t+\epsilon\big)\\ &\approx \frac{\alpha Q(t)}{2}\sin\Big(\big((\omega_n-\omega_c)t\big)\Big) - \frac{\alpha\epsilon Q(t)}{2}\cos\big((\omega_n-\omega_c)t\big)\\ &\quad - \frac{\alpha Q(t)}{2}\sin\big((\omega_n+\omega_c)t\big) - \frac{\alpha\epsilon Q(t)}{2}\cos\big((\omega_n+\omega_c)t\big)\end{aligned}$$

If the quadrature skew ϵ is small enough that $\cos(\epsilon) \approx 1$ and $\sin(\epsilon) \approx \epsilon$, then the terms that contain ϵ can be simplified by applying the formulas for sums and differences of sines and cosines. The approximations are shown in equation (A.7) for those terms.

When the terms are added together, the output of the vector modulator with the presence of amplitude imbalance and quadrature skew is shown as (A.8).

$$\begin{aligned}s(t) &= \left(\frac{I(t)}{2}(1+\alpha) - \frac{Q(t)}{2}\epsilon\alpha\right)\cos\big((\omega_n+\omega_c)t\big)\\ &- \left(\frac{Q(t)}{2}(1+\alpha) + \frac{I(t)}{2}\epsilon\alpha\right)\sin\big((\omega_n+\omega_c)t\big)\\ &- \left(\frac{Q(t)}{2}(1-\alpha) + \frac{\alpha\epsilon I(t)}{2}\right)\sin\big((\omega_n-\omega_c)t\big)\\ &+ \left(\frac{I(t)}{2}(1-\alpha) - \frac{\alpha\epsilon Q(t)}{2}\right)\cos\big((\omega_n-\omega_c)t\big)\end{aligned} \qquad \textbf{(A.8)}$$

If there were no amplitude imbalance, then α would be equal to unity. If there were no quadrature skew, ϵ would be equal to zero. It can be seen that $\alpha=1$ and $\epsilon=0$ substituted into equation (A.8) causes the frequency difference terms to drop out, and the equation reduces to the form shown in (A.5). The presence of amplitude imbalance and quadrature skew causes the appearance of the frequency difference terms. These terms form an undesired image frequency located at ω_n rad/s below the carrier frequency ω_c.

The ratio of power contained in the desired signal, which contains the $(\omega_n+\omega_c)$ terms, to the undesired image, which contains the $(\omega_n-\omega_c)$ terms, is the image rejection ratio, or IRR. The power of each frequency is computed by summing the square of the coefficients of the sines and cosines seen in (A.8) while I(t) and Q(t) are assigned a value of unity.

$$IRR = \frac{\left(\frac{1}{2}(1+\alpha) - \frac{\epsilon\alpha}{2}\right)^2 + \left(\frac{1}{2}(1+\alpha) + \frac{\alpha\epsilon}{2}\right)^2}{\left(\frac{1}{2}(1-\alpha) - \frac{\epsilon\alpha}{2}\right)^2 + \left(\frac{1}{2}(1-\alpha) + \frac{\epsilon\alpha}{2}\right)^2} \qquad \textbf{(A.9)}$$

Carrying out the quadratic binomial expansions and combining like terms in (A.9) yields the equation (A.10).

$$IRR = \frac{1+2\alpha+\alpha^2(1+\epsilon^2)}{1-2\alpha+\alpha^2(1+\epsilon^2)} \qquad \textbf{(A.10)}$$

Observe that with no amplitude imbalance, $\alpha=1$ causes (A.10) to reduce to the expression $(4+\epsilon^2)/\epsilon^2$. With no quadrature skew, $\epsilon=0$ causes the expression for IRR to reduce to $(1+\alpha)^2/(1-\alpha)^2$. If there is no quadrature skew and no amplitude imbalance, the IRR ratio becomes infinite, as expected.

Effect of Digital Frequency on Constellation

The derivation of the image rejection ratio shows how the combination of quadrature skew and amplitude imbalance in the analog circuitry causes an undesired image to be present in the spectrum. Interestingly, if either the upper sideband output or the undesired image is inspected by itself, it can be shown that the constellation is free from the effects of amplitude imbalance and quadrature skew. Consider the desired upper sideband terms of equation (A.8), represented alone in equation (A.11).

$$\begin{aligned}s_{usb}(t) &= \left(\frac{I(t)}{2}(1+\alpha) - \frac{Q(t)}{2}\epsilon\alpha\right)\cos\big((\omega_n+\omega_c)t\big)\\ &- \left(\frac{Q(t)}{2}(1+\alpha) + \frac{I(t)}{2}\epsilon\alpha\right)\sin\big((\omega_n+\omega_c)t\big)\end{aligned} \qquad \textbf{(A.11)}$$

In equation (A.11), the quantity $\cos(\gamma)$ is substituted for $(1+\alpha)/2$, and $\sin(\gamma)$ is substituted for $(\epsilon\alpha/2)$ to produce equation (A.12).

$$\begin{aligned}s_{usb}(t) &= (I(t)\cos(\gamma) - Q(t)\sin(\gamma))\cos\big((\omega_n+\omega_c)t\big)\\ &- (Q(t)\cos(\gamma) + I(t)\sin(\gamma))\sin\big((\omega_n+\omega_c)t\big)\end{aligned} \qquad \textbf{(A.12)}$$

The products of cosines and sines in (A.12) are carried out and decomposed into I(t) and Q(t) components in (A.13).

$$s_{usbI}(t) = I(t)\{\cos(\gamma)\cos((\omega_n + \omega_c)t) - \sin(\gamma)\sin((\omega_n + \omega_c)t)\} \quad \textbf{(A.13)}$$

$$s_{usbQ}(t) = -Q(t)\{\sin(\gamma)\cos((\omega_n + \omega_c)t) + \cos(\gamma)\sin((\omega_n + \omega_c)t)\}$$

The I(t) and Q(t) components of equation (A.13) can be simplified further and recombined as shown in (A.14) using trigonometric identities for the cosines and sines of sums.

$$s_{usb}(t) = I(t)\cos((\omega_n + \omega_c)t + \gamma) - Q(t)\sin((\omega_n + \omega_c)t + \gamma) \quad \textbf{(A.14)}$$

Equation (A.14) shows that on the upper sideband, the amplitude imbalance and quadrature skew factors α and ϵ have been reduced to a common phase offset present on the up-converted carrier. Signal I(t) exists entirely on the cosine and Q(t) entirely on the sine.

A similar derivation applied to the lower sideband terms of (A.8) demonstrates that it also has the I(t) and Q(t) rebalanced and with quadrature skew removed. Extracting the lower sideband components of (A.8) produces equation (A.15).

$$s_{lsb}(t) = -\left(\frac{Q(t)}{2}(1-\alpha) + \frac{\alpha\epsilon I(t)}{2}\right)\sin((\omega_n - \omega_c)t) + \left(\frac{I(t)}{2}(1-\alpha) - \frac{\alpha\epsilon Q(t)}{2}\right)\cos((\omega_n - \omega_c)t) \quad \textbf{(A.15)}$$

This time, the quantity cos(λ) is substituted in place of (1-α)/2, and sin(λ) replaces (εα/2) to produce equation (A.16).

$$s_{lsb}(t) = -(Q(t)\cos(\lambda) + I(t)\sin(\lambda))\sin((\omega_n - \omega_c)t) + (\mathrm{I(t)}\cos(\lambda) - Q(t)\sin(\lambda))\cos((\omega_n - \omega_c)t) \quad \textbf{(A.16)}$$

Separating the I(t) and Q(t) components from (A.16) produces (A.17).

$$s_{lsbI}(t) = I(t)\{\cos(\lambda)\cos((\omega_n - \omega_c)t) - \sin(\lambda)\sin((\omega_n - \omega_c)t)\} \quad \textbf{(A.17)}$$

$$s_{lsbQ}(t) = -Q(t)\{\sin(\lambda)\cos((\omega_n - \omega_c)t) + \cos(\lambda)\sin((\omega_n - \omega_c)t)\}$$

The final step of the proof is to apply formulas for cosines and sines of sums and to recombine the I and Q components into a single expression.

$$s_{lsb}(t) = I(t)\cos((\omega_n - \omega_c)t + \lambda) - Q(t)\sin((\omega_n - \omega_c)t + \lambda) \quad \textbf{(A.18)}$$

As with (A.14) for the upper sideband, equation (A.18) demonstrates that the lower sideband exhibits no quadrature skew or amplitude imbalance.

Effect of Sign of Digital Frequency on Vector Modulator Output

If the complex exponential in (A.1) were changed from positive to negative, then the baseband data would be down-coverted in the digital domain. This change would mean sending I(t)cos(ω_nt)+Q(t)sin(ω_nt) to the COSINE DAC and Q(t)cos(ω_nt) – I(t)sin(ω_nt) to the SINE DAC. With no quadrature skew or amplitude imbalance, the frequency difference terms (ω_n-ω_c) would combine and the frequency sum terms (ω_n-ω_c) would cancel. The presence of amplitude imbalance or quadrature skew would cause the presence of an image at (ω_n+ω_c).

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